Transition Dynamics of a Mass Deportation*

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Abstract

In discussions about immigration, the possibility of deporting the whole population of illegal immigrants is often bandied about. Most economists, and probably most people, intuit that this would be a bad idea, but rigorous arguments are difficult to find. Here we construct a simple three-period overlapping-generations model with high- and low-educated workers. Both types are ex ante identical. Upgrading education from low to high is costly in resources, time, and utility. Illegal immigrants are assumed to be subsumed within the class of low-educated workers. We study the transition dynamics following the deportation of a large fraction of low-educated workers. In the long run, the economy returns to the original intrinsic equilibrium, albeit with a smaller population and GDP. The elasticity of low-educated wages with respect to the supply of low-educated labor is the output share of expenditures on capital and high-educated labor divided by the elasticity of substitution between low-educated labor and capital/high-educated labor expenditures. If low-educated labor is a substitute for capital and high-educated labor, this will be less than one and often much less than one, so a deportation of low-educated labor has a negligible effect on low-educated wages and the welfare of low-educated households. High-educated households see a reduction in income. Whether the deportation has widespread effects on the economy depends on patience. If the population of young who initially planned to get high education resist the temptation to drop out of school, only the initial cohort of high-educated workers will be hurt by the deportation since their wages will go down. If a sufficiently large number of this group do leave school, the resulting output loss next period will lead the economy to alternate between good and bad periods.

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Although immigration is a subject of continual political debate, much of the discussion is often confused. For example, people who claim to be proponents of free markets are often the most vocal opponents of immigration reform. Some of this confusion can be attributed to the lack of a simple model that addresses concerns about immigration. Most economists–and probably most people–intuit that deporting the whole

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population of illegal immigrants would hurt others besides the immigrants. However, one cannot easily rebut those who believe immigrants are responsible for most of their troubles with the same rigor as one can defend laws against monopolies. Here we present a model that, while more complicated than a basic supply and demand model, is stripped down to include only the essential elements for quantifying the extent to which immigration benefits or hurts different groups.

The primary economic concern related to immigrants stems from their impact on factor prices.[1] In a representative-agent economy with homogenous labor, immigration would have no effect on wages (or interest rates) if immigrants have the same preferences and, therefore, the same propensity to save as natives. Therefore, heterogeneous labor is an essential element for modeling immigration. Here we take it as axiomatic that there is no intrinsic difference between immigrants and natives. Given the same choice set the two populations will realize the same outcomes.

Our model has two types of workers: high-educated and low-educated. Both types are born identical. Obtaining more education is costly in time, resources, and utility. Given an infinite lifespan, everyone should eventually seek higher education, so we impose an overlapping-generations structure to explain why higher education is not an absorbing state. A minimum lifespan of three periods is needed so a high-educated household has time to get educated and work, and also has a retirement period to save for. This last period is essential since most capital belongs to high-educated households who would otherwise have no motive to save. An adult lifespan of 60 years from ages 20 to 80 implies a period length of twenty years. Note that twenty years of schooling is consistent with the amount of training required for medical school or to complete a doctorate but not for a basic college degree. Whereas most research that distinguishes between skilled and unskilled sets a bachelor's degree as the dividing threshold, we set the threshold as a doctoral degree in any field.

We assume the population of illegal immigrants subject to deportation falls within the population of low-educated households. Lumping illegal immigrants into the same household type as households with a college degree but no further education may seem unrealistic, but the gap in wages between a lawyer or physician and a person with just a college degree is much larger than the gap in wages between a person with just a college degree and a person who never went to college. Our principal exercise is then to consider the transition dynamics that follow the deportation of a large fraction of low-educated households. In the immediate aftermath of the deportation, GDP will fall because of the shock to labor supply. The resulting change in factor prices will alter the calculus by which young people decide whether to invest in high education. The nontrivial question is how this change in the skill composition propagates into the future.

This paper departs from the existing literature by focusing on the effects of removing workers from the economy whereas previous research, such as Ben-Gad (2005, 2008) and Genc (2015), has focused on the consequences of letting more workers in.[2] Our model is most similar to Boldrin and Montes (2015), and our experiment is the flip side of their experiment. When studying transition dynamics, there is a fundamental asymmetry between accepting more immigrants and sending immigrants "home". To understand how more immigrants affect the economy over time it is necessary to model the process by which immigrants are assimilated into the native population, and quantitative results will depend sensitively on the nature of this

[1] An often-cited secondary concern is that they are a drain on the economy because they suck up too many resources from government programs. (Storesletten (2003) computed the fiscal impact of a typical immigrant for Sweden, a state with quite generous benefits.) However, most claims of this sort are based on naive partial-equilibrium economics. For the United States at least, simple back-of-the-envelope calculations show that immigrants, both legal and illegal, contribute far more to GDP than what they consume in government spending. Illegal immigrants are not eligible to collect benefits from Social Security the largest entitlement program in the US.

In the same vein, it is often argued that illegal immigrants do not pay their fair share for government services since they do not pay income taxes. However, it would be much less burdensome to modify the tax structure by putting more weight on less avoidable taxes, such as sales taxes, than it would be to deport illegal immigrants.

[2] German Cubas says there is work based on the Roy model that comes to very different conclusions.

assimilation process.[3] On the other hand, asking how a mass deportation will affect the economy over time is much simpler because we can just assume the emigrés were already assimilated before their departure. This does not break from our premise that immigrants and natives are ex ante identical because paying for education in the model requires access to credit markets. If we assume that illegal immigrants are denied access to credit, they will be stuck in the low-educated population while natives and legal immigrants are free to choose their type.

Much of the discussion of deporting illegal immigrants seems to be premised on the assumption that this will have a huge impact on the wages of low-educated households who remain behind. We show for a general constant-returns-to scale, strictly concave production function involving the three inputs of capital, low-educated labor, and high-educated labor that the elasticity of the low-educated wage with respect to the low-educated labor supply is the output share of expenditures on capital and high-educated labor divided by the elasticity of substitution between these expenditures and low-educated labor. The share of capital and high-educated labor expenditures is easily pinned down to 0.3-0.4 by NIPA data for the United States. It is also clear that low-educated labor must be a substitute for high-educated labor and capital, or else *Homo sapiens* could not have produced the first capital. Thus the observable properties of the production function set an upper bound for the elasticity of low-educated wages with respect to low-educated labor that is significantly below one. A deportation of 5% of the low-educated population will raise low-educated wages by much less than 5%. Accounting for the loss in capital and the shifting of young households from high education to low education will further reduce this increase. The welfare gains of low-educated households from a mass deportation will be much smaller than is often supposed. Meanwhile the brunt of the loss of output must be borne by someone. Since they much fewer in number, the high-educated households will suffer a welfare loss an order of magnitude larger than than the gain by low-educated households.

Immediately after a deportation shock, the fraction of middle-aged households with low education will be smaller than expected, so the ratio of high-educated to low-educated labor will increase. This lowers the skill premium, which hurts high-educated, middle-aged households while benefiting low-educated, middle-aged households. Since the gains from education are smaller, a larger than expected fraction of young households will enter the labor force immediately.[4] What happens next depends on how many young households opt against higher education. Both the wages of a low-educated household at the time of the deportation shock and the wages of a high-educated household one period later will increase. The change in low-educated wages will typically be an order of magnitude smaller than the change in high-educated wages. Nevertheless, if households are sufficiently impatient, they may still respond more to the latter.

If the next period's pool of high-educated labor is only marginally depleted, the negative welfare effects of the deportation shock will be confined to high-educated workers at the time of the shock. The deportation shock removes households with low amounts of capital, so the capital to labor ratio increases. Since there are fewer low-educated households, the higher ratio of high- to low-educated labor and the higher capital to labor ratio together yield a larger output per capita. Because low-educated households can save more than they could before the shock, the capital to labor ratio stays above its steady-state level. The ratio of high-educated to low-educated labor falls in future periods, but in this case that effect is small, so output per capita remains above its pre-shock level as it returns to its steady state value. Thus, everyone born after the shock is better off.

If, on the other hand, next period's pool of high-educated labor is depleted enough so output per capital will fall next period, the negative welfare effects of the deportation shock will impact (some) future generations of both types. In this case, the response of variables in the period of the shock is qualitatively the same as in the previous case. However, the situation for households born the period after the shock is

[3]For example, Boldrin and Montes (2015) assume that immigrants die without assimilating, but their children are natives. Ben-Gad (2005), in contrast, has a dynasty model in which immigrant families gradually acquire human capital.

[4]Jackson (2014) has shown empirically that the educational mix of immigrants does affect the educational choices of natives. An influx of unskilled immigrants will spur more natives to go to college. Here we assume the opposite is also true.

very different from the previous case. Whereas the ratio of high- to low-educated households was higher than the steady-state value in the period of the shock, it swings the other way in the period after the shock. High-educated workers in this period will all be middle-aged. Born in the period of the shock, they will gain from the higher wages they earn now, just as their low-educated counterparts gained from the higher wages they earned the previous period. But low-educated wages in this next period will be lower than in the steady state. Thus low-educated households born in this next period will be worse off than in the steady state because of these lower wages. The low wages will also induce more of these young households to acquire high education, resulting in a higher than normal ratio of high- to low-educated workers in the next ensuing period. As it was for the workers born at the time of the shock, this higher ratio will yield a wage cycle that is advantageous for the households born two periods after the shock while it hurts the households born one period after the shock. More households from this second-period cohort will decide not to get educated, and the cycle will repeat with damped oscillations till the economy returns to the steady state. Households born in even-number periods after the shock will benefit while households born in odd-number periods will be hurt.

There is actually one odd case where the common intuition that the initial generation of low-educated workers will enjoy substantial welfare gains after a mass deportation does, in fact, come true. This is likely an artifact of the stylized assumption that households only live for three periods and will likely disappear in more fine-grained overlapping-generations models. However, for some parameters there are multiple steady-state equilibria. In addition to the usual equilibrium where all variables converge to a steady-state limit, there may also be two-cycle equilibria where the economy alternates between periods when almost nobody acquires high education and periods when a sizable chunk of the population gets educated. A deportation can cause a transition from the constant steady-state equilibrium to the two-cycle equilibrium. The deportation is really just a catalyst or sunspot and has no direct link to the welfare gains of the initial cohort of low-educated, middle-aged workers. What causes their wages to go up is the decision of so many young people to get educated, which is triggered by the deportation but not explained by the deportation. The two-cycle equilibrium is in fact a poverty trap caused by too much investment in human capital at the expense of underinvestment in physical capital. Young people get educated in large numbers because of a coordination failure. They anticipate interest rates will be negative after they retire, which is a consequence of so many of them getting educated and accumulating too much capital when they later hit the labor force. It is rational for so many of them to get education because this is necessary to pay for a retirement when interest rates are negative because they will accumulate so much capital. During the alternating periods, meanwhile, those few young people who get educated enjoy a gigantic skill premium while the many workers who do not get educated enjoy the high wages resulting from the high capital stock. Saving for retirement is not so costly for those born in the alternating periods since they will retire when the capital stock is low and interest rates are high, so they are not under so much pressure to get educated. However, the gains of the cohorts born in good periods are far outweighed, by several orders of magnitude, by the losses of the cohorts born in bad periods.

The paper is organized as follows. The model is described in Section 1. The elasticity of low-educated wages with respect to low-educated labor is computed for a general production function in Section 2. The baseline model is calibrated to match the existing US economy in Section 3. The consequences of a mass deportation of low-educated workers are discussed in Section 4. The consequences of a mass emigration of high-educated workers are discussed in Section **??**. Section 6 offers concluding remarks.

1 The Model

At time t, a measure N_t of households is born that solve

$$U_t = \max_{\{c_{t,0};c_{t+1,1};c_{t+2;2}\},e_t\in\{0,1\}} u(c_{t,0};\gamma) + \beta u(c_{t+1,1};\gamma) + \beta^2 u(c_{t+2,2};\gamma) - \rho e_t \tag{1}$$

subject to

$$c_{t,0} + b_{t+1,1} = w_t(e_t)(1 - e_t) - \tau e_t \tag{2}$$

$$c_{t+1,1} + b_{t+2,2} = w_{t+1}(e_t) + R_{t+1} b_{t+1,1} \tag{3}$$

$$c_{t+2,2} = R_{t+2} b_{t+2,2}. \tag{4}$$

$$b_{t+1,1} \geq -(1+\chi)\tau e_t.$$

The discount factor is $\beta > 0$. Period utility is CRRA with inverse elasticity of substitution γ:

$$u(c;\gamma) = \begin{cases} \ln c & \gamma = 1 \\ \frac{1}{1-\gamma} c^{1-\gamma} & \gamma \neq 1 \end{cases}. \tag{5}$$

The household chooses at age 0 whether to invest in education. If the household chooses $e_t = 0$ and does not invest in education, it begins work right away and also works in middle age, earning the wage $w_s(0)$ at time s. If the household chooses $e_t = 1$, it goes to school at age 0, which costs both tuition τ and a utility cost $\rho \geq 0$. Households are only allowed to borrow when young to pay for education and some amount of consumption.[5] The allowed ratio of consumption to tuition is $\chi > 0$.

At age 1, the educated household earns the wage $w_{t+1}(1)$. Households can save or borrow at the gross interest rate R_{t+1} at time t. For now we assume perfect capital markets, but we will undoubtedly need to introduce borrowing frictions to account for the different consumption behavior of educated and uneducated households.

Let $n_t(0)$ represent the population of households that choose not to get educated at age t and $n_t(1)$ the population that choose to get educated at age t, where

$$n_t(0) + n_t(1) = N_t. \tag{6}$$

Unskilled labor at t is

$$N_t(0) = n_t(0) + n_{t-1}(0) \tag{7}$$

while skilled labor at t is

$$N_t(1) = n_{t-1}(1). \tag{8}$$

For the household variable x_t, let $x_t(0)$ be the choice of an unskilled household while $x_t(1)$ is the choice of a skilled household. Then the capital stock at time t is

$$K_t = n_{t-1}(0)b_{t,1}(0) + n_{t-1}(1)b_{t,1}(1) + n_{t-2}(0)b_{t,2}(0) + n_{t-2}(1)b_{t,2}(1), \tag{9}$$

which depreciates at the rate $\delta \in [0, 1]$. The production function is

$$Y_t = F(K_t, N_t(0), N_t(1)), \tag{10}$$

[5] This borrowing constraint is needed to explain why high- and low-educated households hold differing amounts of capital relative to income. This happens in the model because high-educated households have limited consumption while young, so they have much higher consumption while middle-aged and old, which means they must save more for retirement. Low-educated households, in contrast, have relatively smooth consumption profiles.

where F exhibits constant returns to scale. Firms behave competitively so factor prices are

$$R_t = F_K(K_t, N_t(0), N_t(1)) + 1 - \delta \tag{11}$$

$$w_t(0) = F_{N(0)}(K_t, N_t(0), N_t(1)) \tag{12}$$

$$w_t(1) = F_{N(1)}(K_t, N_t(0), N_t(1)). \tag{13}$$

There is no intrinsic difference between different types of workers. In equilibrium $U_t(0) = U_t(1)$.

The experiment we are interested in is one where at time 0 the economy is in a steady state. Between periods $t = -1$ and $t = 0$, unskilled workers are removed from the economy in a forced deportation, so $n_{-1}(0)$ and $n_{-2}(0)$ are reduced by the fraction $\zeta \in (0, 1)$.[6] Moreover N_t is also reduced by the fraction ζ thereafter. The capital belonging to removed workers also disappears from the economy.[7] What do the resulting equilibrium dynamics look like?[8]

We use the following solution algorithm. We guess at a sequence of capital stocks $\{K_t\}_{t=0}^{\infty}$ and a sequence of occupation choices $\{n_t(0), n_t(1)\}_{t=0}^{\infty}$ that satisfy the population constraint (6). This is enough information to compute the whole sequence of factor prices. We can then solve the household's problem at each $t \geq 0$ for both types of workers. We then update $\{K_t\}_{t=0}^{\infty}$ based on these choices and adjust $n_t(0)$ and $n_t(1)$ based on the difference

$$\Delta U_t = U_t(1) - U_t(0). \tag{14}$$

An equilibrium will be a fixed point of the mapping from $\{K_t^{(m)}, n_t^{(m)}(0), n_t^{(m)}(1)\}_{t=0}^{\infty}$ to $\{K_t^{(m+1)}, n_t^{(m+1)}(0), n_t^{(m+1)}(1)\}_{t=0}^{\infty}$.

The immediate effect of the deportation will be to reduce unskilled labor and capital, though the change in capital is likely to be small since skilled households will presumably hold the bulk of the capital. Both effects should reduce the skilled wage. The return on capital is likely to fall too, although this will depend on the calibration.

In calibrating the model, the skilled labor should really be viewed as those who went to graduate or professional school instead of those who simply went to college. Given that most unskilled labor begins around age 15 and I finished graduate school at age 33, that would be consistent with a 20-year period. Going forward, the income difference between those who have an advanced degree and those who only go to college is likely to be more than the income difference between those who simply go to college and those who do not, so it is reasonable to lump all workers without an advanced degree together in a first crack at this type of model. Going forward with more than 3 periods we could disaggregate the unskilled.

Since

$$b_{t+2,2} = \frac{c_{t+2,2}}{R_{t+2}}$$

and

$$b_{t+1,1} = \frac{c_{t+1,1} - w_{t+1}(e_t)}{R_{t+1}} + \frac{c_{t+2,2}}{R_{t+1} R_{t+2}},$$

[6] We can also use the model to simulate what would happen if a large fraction of high-educated workers left the economy as in *Atlas Shrugged*.

[7] This distinguishes a deportation from a holocaust in which the people are removed but their property is confiscated. Since capital per worker will increase, the remaining population will be better off at least in the short run, though the standard of living will likely return to its initial, steady state value in the long run. It is GDP, however, that translates into geopolitical power, not GDP per capita, and GDP will fall with the labor supply. This likely is an important part of why the Germans lost World War II and why, historically, successful societies have usually assimilated conquered populations instead of obliterating them.

[8] For simplicity we assume households cannot switch from being skilled to unskilled at age 1 because they have not had the training of an unskilled worker during the first period. I suspect, though, that even if we allowed them to make this choice the continuation utility of a skilled household will always be higher than the continuation utility of an unskilled household unless χ is very large.

the lifecycle budget constraint is

$$c_{t,0} + \frac{c_{t+1,1}}{R_{t+1}} + \frac{c_{t+2,2}}{R_{t+1}R_{t+2}} = w_t(e_t)(1-e_t) + \frac{w_{t+1}(e_t)}{R_{t+1}} - \tau e_t. \tag{15}$$

Define

$$H_t(0) = w_t(0) + \frac{w_{t+1}(0)}{R_{t+1}} \tag{16}$$

and

$$H_t(1) = \frac{w_{t+1}(1)}{R_{t+1}} - \tau \tag{17}$$

as the human wealth of an agent born at t, so the constraint is

$$c_{t,0} + \frac{c_{t+1,1}}{R_{t+1}} + \frac{c_{t+2,2}}{R_{t+1}R_{t+2}} = H_t.$$

The household's problem has the Lagrangian

$$\begin{aligned} L_t &= u(c_{t,0};\gamma) + \beta u(c_{t+1,1};\gamma) + \beta^2 u(c_{t+2,2};\gamma) \\ &\quad + \lambda_t \left[H_t - c_{t,0} - \frac{c_{t+1,1}}{R_{t+1}} - \frac{c_{t+2,2}}{R_{t+1}R_{t+2}} \right]. \end{aligned}$$

The first-order conditions are

$$\begin{aligned} \frac{\partial L_t}{\partial c_{t,0}} &= c_{t,0}^{-\gamma} - \lambda_t = 0 \\ \frac{\partial L_t}{\partial c_{t+1,1}} &= \beta c_{t+1,1}^{-\gamma} - \frac{\lambda_t}{R_{t+1}} = 0 \\ \frac{\partial L_t}{\partial c_{t+2,2}} &= \beta^2 c_{t+2,2}^{-\gamma} - \frac{\lambda_t}{R_{t+1}R_{t+2}} = 0 \end{aligned}$$

Thus we get the Euler equations

$$c_{t+1,1} = (\beta R_{t+1})^{\frac{1}{\gamma}} c_{t,0} \tag{18}$$

$$c_{t+2,2} = (\beta R_{t+2})^{\frac{1}{\gamma}} c_{t+1,1} \tag{19}$$

Let us define

$$\phi_t = (\beta R_t^{1-\gamma})^{-\frac{1}{\gamma}} \tag{20}$$

so

$$\frac{c_{t+1,1}}{R_{t+1}} = (\beta R_{t+1}^{1-\gamma})^{\frac{1}{\gamma}} c_{t,0} = \frac{c_{t,0}}{\phi_{t+1}}$$

and

$$\frac{c_{t+2,2}}{R_{t+1}} = (\beta R_{t+2}^{1-\gamma})^{\frac{1}{\gamma}} c_{t+1,1} = \frac{c_{t+1,1}}{\phi_{t+2}}$$

Inserting these in the lifetime budget constraint,

$$\left[1 + \frac{1}{\phi_{t+1}} + \frac{1}{\phi_{t+1}\phi_{t+2}}\right] c_{t,0} = H_t$$

Consumptions are

$$c_{t,0} = \frac{H_t}{1 + \frac{1}{\phi_{t+1}} + \frac{1}{\phi_{t+1}\phi_{t+2}}} \tag{21}$$

$$c_{t+1,1} = (\beta R_{t+1})^{1/\gamma} \frac{H_t}{1 + \frac{1}{\phi_{t+1}} + \frac{1}{\phi_{t+1}\phi_{t+2}}} \tag{22}$$

$$c_{t+2,2} = (\beta^2 R_{t+1} R_{t+2})^{1/\gamma} \frac{H_t}{1 + \frac{1}{\phi_{t+1}} + \frac{1}{\phi_{t+1}\phi_{t+2}}}. \tag{23}$$

2 Immediate Effect of Deportation on Low-Educated Wages

The main argument for why a deportation of illegal immigrants would be beneficial is that it will lead to higher wages for the low-educated households that remain. We can obtain some insightful theoretical results about the magnitude of this change in wages without making assumptions about the production function. Suppose we just have two inputs Z_1 and Z_2 with factor prices p_1 and p_2, and the production function is $H(Z_1, Z_2)$. Then in equilibrium

$$p_i = H_i(Z_1, Z_2).$$

$$\frac{\partial p_2}{\partial z_2} = H_{22}(Z_1, Z_2). \tag{24}$$

If the production function has constant returns to scale

$$H(Z_1, Z_2) = Z_2 h(z),$$

where

$$h(z) = H(z, 1)$$

and

$$z = \frac{Z_1}{Z_2}.$$

$$\begin{aligned} p_1 &= h'(z) \\ p_2 &= h(z) - zh'(z) \end{aligned}$$

$$\frac{p_2}{p_1} = \frac{h(z)}{h'(z)} - z$$

$$\frac{\partial \left(\frac{p_2}{p_1}\right)}{\partial z} = \frac{h'(z)}{h'(z)} - \frac{h(z)h''(z)}{(h'(z))^2} - 1 = -\frac{h(z)h''(z)}{(h'(z))^2}$$

The elasticity of substitution between the two factors is given by

$$\xi(z)^{-1} = \frac{\partial \ln \left(\frac{p_2}{p_1}\right)}{\partial \ln z} = -\frac{h(z)h''(z)}{(h'(z))^2} \frac{z}{\frac{h(z)}{h'(z)} - z} = -\frac{zh''(z)}{h'(z)} \frac{h(z)}{h(z) - h'(z)z}, \tag{25}$$

and

$$\alpha(z) = \frac{zh'(z)}{h(z)} \tag{26}$$

is the output share of factor 1.

$$H_{22}(Z_1, Z_2) = h'(z)\left(-\frac{Z_1}{Z_2^2}\right) - h'(z)\left(-\frac{Z_1}{Z_2^2}\right) - zh''(z)\left(-\frac{Z_1}{Z_2^2}\right) = \frac{z^2}{Z_2}h''(z)$$

$$\begin{aligned}
\frac{\partial \ln(p_2)}{\partial \ln(Z_2)} &= \frac{Z_2}{p_2}H_{22}(Z_1, Z_2) = \frac{Z_2}{h(z) - zh'(z)}\left(\frac{z^2}{Z_2}h''(z)\right) = \frac{z^2 h''(z)}{h(z) - zh'(z)} \\
&= \frac{zh''(z)}{h(z) - zh'(z)}z \\
&= -\left(-\frac{zh''(z)}{h'(z)}\frac{h(z)}{h(z) - h'(z)z}\right)\frac{zh'(z)}{h(z)} \\
&= -\frac{\alpha(z)}{\xi(z)}.
\end{aligned}$$

Thus the elasticity of the price of factor 2 with respect to the supply of factor 2 is the output share of factor 1 divided between the elasticity of substitution between the two factors.

We can apply this to the production function (10) if we write treat low-educated labor as factor 2 and construct a composite factor from high-educated labor and capital to serve as factor 1.

The firm's problem is

$$\max F(K, N(0), N(1)) - \widetilde{r}K - w(0)N(0) - w(1)N(1),$$

where $\widetilde{r} = r + \delta$ is the gross return on capital. Let us define expenditures on capital and high-skilled labor as

$$E = \widetilde{r}K + w(1)N(1).$$

Then we can redefine the production function in terms of low-educated labor and capital/high-skilled labor expenditures:

$$G(E, N(0)) = \max_{K,N(1)} F(K, N(0), N(1)) \tag{27}$$

subject to

$$\widetilde{r}K + w(1)N(1) = E.$$

Then we can rewrite the firm's problem as

$$\max G(E, N(0)) - w(0)N(0) - E.$$

Suppose that F exhibits constant returns to scale. Then for $\lambda > 0$,

$$G(\lambda E, \lambda N(0)) = \max_{K,N(1)} F(K, \lambda N(0), N(1))$$

subject to

$$\widetilde{r}K + w(1)N(1) = \lambda E.$$

Thus

$$G(\lambda E, \lambda N(0)) = \max_{K,N(1)} \lambda F\left(\frac{K}{\lambda}, N(0), \frac{N(1)}{\lambda}\right)$$

subject to

$$\widetilde{r}\frac{K}{\lambda} + w(1)\frac{N(1)}{\lambda} = E.$$

Let $K_\lambda = \frac{K}{\lambda}$ and $N_\lambda(1) = \frac{N(1)}{\lambda}$. Then

$$G(\lambda E, \lambda N(0)) = \max_{K_\lambda, N_\lambda(1)} \lambda F\left(K_\lambda, N(0), N_\lambda(1)\right)$$

subject to

$$\widetilde{r} K_\lambda + w(1) N_\lambda(1) = E.$$

But K_λ and $N_\lambda(1)$ are dummy variables, so

$$G(\lambda E, \lambda N(0)) = \max_{K, N(1)} \lambda F\left(K, N(0), N(1)\right)$$

subject to

$$\widetilde{r} K + w(1) N(1) = E.$$

Thus

$$G(\lambda E, \lambda N(0)) = \lambda G(E, N(0)).$$

Since G has constant returns to scale, we can employ the above result to show that the elasticity of low-educated waged with respect to low-educated labor is the share of capital-skill expenditures divided by the elasticity of substitution between low-educated labor and capital-skill expenditures.

3 Calibration of Baseline Model

Our calibration targets are 0.8 for the elasticity of substitution between capital and skilled labor and 4.0 for the elasticity between capital/skilled labor and unskilled labor to 4.0.[9]

Let $A_{KX}, A_{SX}, A_{XY}, A_{UY} \geq 0$ be productivity indices for capital, skilled labor, and unskilled labor.

$$X_t = \left[A_{KX} K_t^{\sigma_{KS}} + A_{SX} N_t(1)^{\sigma_{KS}}\right]^{\frac{1}{\sigma_{KS}}} \tag{28}$$

is the capital-skill aggregator where $A_{KX} + A_{SX} = 1$, $\sigma_{KS} < 1$ and

$$\xi_{KS} = \frac{1}{1 - \sigma_{KS}} \tag{29}$$

is the elasticity of substitution between capital and skilled labor. The production function is

$$Y_t = \left[A_{XY} X_t^{\sigma_{XU}} + A_{UY} N_t(0)^{\sigma_{XU}}\right]^{\frac{1}{\sigma_{XU}}}, \tag{30}$$

where $A_{XY} + A_{UY} = 1$, $\sigma_{XU} < 1$ and

$$\xi_{XU} = \frac{1}{1 - \sigma_{XU}}. \tag{31}$$

Since $A_{SX} = 1 - A_{KX}$ and $A_{UY} = 1 - A_{XY}$, the production function has two remaining parameters A_{KX} and A_{XY}, which can be calibrated to match the share of capital and the skill premium. The capital-output ratio will also be important for pinning down the preference parameters.

$$\frac{\partial X_t}{\partial K_t} = \frac{1}{\sigma_{KS}} \left[A_{KX} K_t^{\sigma_{KS}} + A_{SX} N_t(1)^{\sigma_{KS}}\right]^{\frac{1}{\sigma_{KS}} - 1} \sigma_{KS} A_{KX} K_t^{\sigma_{KS} - 1}$$

[9] Note that these elasticities were estimated under the definitions that skilled labor means labor with a college degree and unskilled labor means labor without a college degree. We are using different definitions of labor, so ideally the elasticities should be reestimated. That said, the results of Section 2 show that the qualitative results will not be very sensitive to these estimates.

$$\frac{\partial X_t}{\partial K_t} = \frac{1}{\sigma_{KS}} \left[A_{KX} K_t^{\sigma_{KS}} + (1 - A_{KX}) N_t(1)^{\sigma_{KS}}\right]^{\frac{1-\sigma_{KS}}{\sigma_{KS}}} \sigma_{KS} A_{KX} K_t^{\sigma_{KS}-1}$$

$$\frac{\partial X_t}{\partial K_t} = A_{KX} X_t^{1-\sigma_{KS}} K_t^{\sigma_{KS}-1}$$

Thus

$$\frac{\partial X_t}{\partial K_t} = A_{KX} \left(\frac{X_t}{K_t}\right)^{1-\sigma_{KS}} \tag{32}$$

Similiarly,

$$\frac{\partial X_t}{\partial N_t(1)} = (1 - A_{KX}) \left(\frac{X_t}{N_t(1)}\right)^{1-\sigma_{KS}} \tag{33}$$

Likewise,

$$\frac{\partial Y_t}{\partial X_t} = A_{XY} \left(\frac{Y_t}{X_t}\right)^{1-\sigma_{XU}} \tag{34}$$

and

$$\frac{\partial Y_t}{\partial N_t(0)} = (1 - A_{XY}) \left(\frac{Y_t}{N_t(0)}\right)^{1-\sigma_{XU}} \tag{35}$$

Thus

$$w_t(0) = \frac{\partial Y_t}{\partial N_t(0)} = (1 - A_{XY}) \left(\frac{Y_t}{N_t(0)}\right)^{1-\sigma_{XU}} \tag{36}$$

$$w_t(1) = \frac{\partial Y_t}{\partial N_t(1)} = \frac{\partial Y_t}{\partial X_t}\frac{\partial X_t}{\partial N_t(1)} = A_{XY}(1 - A_{KX}) \left(\frac{X_t}{N_t(1)}\right)^{1-\sigma_{KS}} \left(\frac{Y_t}{X_t}\right)^{1-\sigma_{XU}} \tag{37}$$

$$R_t = \frac{\partial Y_t}{\partial K_t} + 1 - \delta = \frac{\partial Y_t}{\partial X_t}\frac{\partial X_t}{\partial K_t} + 1 - \delta$$

$$R_t = A_{KX} A_{XY} \left(\frac{X_t}{K_t}\right)^{1-\sigma_{KS}} \left(\frac{Y_t}{X_t}\right)^{1-\sigma_{XU}} + 1 - \delta \tag{38}$$

The skill premium is

$$\frac{w_t(1)}{w_t(0)} = \frac{A_{XY}(1 - A_{KX}) \left(\frac{X_t}{N_t(1)}\right)^{1-\sigma_{KS}} \left(\frac{Y_t}{X_t}\right)^{1-\sigma_{XU}}}{(1 - A_{XY}) \left(\frac{Y_t}{N_t(0)}\right)^{1-\sigma_{XU}}}$$

$$\frac{w_t(1)}{w_t(0)} = \frac{A_{XY}}{1 - A_{XY}}(1 - A_{KX}) \left(\frac{X_t}{N_t(1)}\right)^{1-\sigma_{KS}} \left(\frac{N_t(0)}{X_t}\right)^{1-\sigma_{XU}}$$

The capital to aggregator ratio is

$$\frac{K_t}{X_t} = \frac{K_t}{\left[A_{KX} K_t^{\sigma_{KS}} + (1 - A_{KX}) N_t(1)^{\sigma_{KS}}\right]^{\frac{1}{\sigma_{KS}}}} = \left[A_{KX} + (1 - A_{KX}) \left(\frac{K_t}{N_t(1)}\right)^{-\sigma_{KS}}\right]^{-\frac{1}{\sigma_{KS}}} \tag{39}$$

The aggregator to output ratio is

$$\frac{X_t}{Y_t} = \frac{X_t}{\left[A_{XY} X_t^{\sigma_{XU}} + (1 - A_{XY}) N_t(0)^{\sigma_{XU}}\right]^{\frac{1}{\sigma_{XU}}}} = \left[A_{XY} + (1 - A_{XY}) \left(\frac{X_t}{N_t(0)}\right)^{-\sigma_{XU}}\right]^{-\frac{1}{\sigma_{XU}}} \tag{40}$$

Thus the capital-output ratio is

$$\frac{K_t}{Y_t} = \left[A_{KX} + (1-A_{KX})\left(\frac{K_t}{N_t(1)}\right)^{-\sigma_{KS}}\right]^{-\frac{1}{\sigma_{KS}}} \left[A_{XY} + (1-A_{XY})\left(\frac{X_t}{N_t(0)}\right)^{-\sigma_{XU}}\right]^{-\frac{1}{\sigma_{XU}}} \tag{41}$$

The share of capital is

$$\alpha_t = \frac{(r_t+\delta)K_t}{Y_t} = A_{KX}A_{XY}\left(\frac{X_t}{K_t}\right)^{1-\sigma_{KS}}\left(\frac{Y_t}{X_t}\right)^{1-\sigma_{XU}}\frac{K_t}{Y_t}$$

$$\begin{aligned}\alpha_t &= A_{KX}A_{XY}\left[A_{KX} + (1-A_{KX})\left(\frac{K_t}{N_t(1)}\right)^{-\sigma_{KS}}\right]^{\frac{1-\sigma_{KS}}{\sigma_{KS}}}\left[A_{XY} + (1-A_{XY})\left(\frac{X_t}{N_t(0)}\right)^{-\sigma_{XU}}\right]^{\frac{1-\sigma_{XU}}{\sigma_{XU}}} \\ &\times\left[A_{KX} + (1-A_{KX})\left(\frac{K_t}{N_t(1)}\right)^{-\sigma_{KS}}\right]^{-\frac{1}{\sigma_{KS}}}\left[A_{XY} + (1-A_{XY})\left(\frac{X_t}{N_t(0)}\right)^{-\sigma_{XU}}\right]^{-\frac{1}{\sigma_{XU}}}\end{aligned}$$

$$\alpha_t = A_{KX}A_{XY}\left[A_{KX} + (1-A_{KX})\left(\frac{K_t}{N_t(1)}\right)^{-\sigma_{KS}}\right]^{-1}\left[A_{XY} + (1-A_{XY})\left(\frac{X_t}{N_t(0)}\right)^{-\sigma_{XU}}\right]^{-1}$$

In the limit as $\sigma_{KS} = \sigma_{XU} = 0$, production is Cobb-Douglas, and the share of capital simplifies to

$$\alpha = A_{KX}A_{XY}$$

as it should.

Since

$$\begin{aligned}Y_t &= (r_t+\delta)K_t + w_t(0)N_t(0) + w_t(1)N_t(1), \\ 1 &= \frac{(r_t+\delta)K_t}{Y_t} + \frac{w_t(0)N_t(0) + w_t(1)N_t(1)}{Y_t} \\ 1-\alpha_t &= \frac{w_t(0)N_t(0) + w_t(1)N_t(1)}{Y_t} \\ w_t(0) &= (1-A_{XY})\left(\frac{Y_t}{N_t(0)}\right)^{1-\sigma_{XU}} \\ \frac{w_t(0)N_t(0)}{Y_t} &= (1-A_{XY})\left(\frac{Y_t}{N_t(0)}\right)^{-\sigma_{XU}}\end{aligned} \tag{42}$$

$$\begin{aligned}w_t(1) &= A_{XY}(1-A_{KX})\left(\frac{X_t}{N_t(1)}\right)^{1-\sigma_{KS}}\left(\frac{Y_t}{X_t}\right)^{1-\sigma_{XU}} \\ &= A_{XY}(1-A_{KX})\left(\frac{X_t}{N_t(1)}\right)^{-\sigma_{KS}}\left(\frac{Y_t}{X_t}\right)^{-\sigma_{XU}}\frac{X_t}{N_t(1)}\frac{Y_t}{X_t} \\ &= A_{XY}(1-A_{KX})\left(\frac{X_t}{N_t(1)}\right)^{-\sigma_{KS}}\left(\frac{Y_t}{X_t}\right)^{-\sigma_{XU}}\frac{Y_t}{N_t(1)}\end{aligned}$$

$$\frac{w_t(1)N_t(1)}{Y_t} = A_{XY}(1-A_{KX})\left(\frac{X_t}{N_t(1)}\right)^{-\sigma_{KS}}\left(\frac{Y_t}{X_t}\right)^{-\sigma_{XU}}$$

Thus

$$(1-A_{XY})\left(\frac{Y_t}{N_t(0)}\right)^{-\sigma_{XU}} + A_{XY}(1-A_{KX})\left(\frac{X_t}{N_t(1)}\right)^{-\sigma_{KS}}\left(\frac{Y_t}{X_t}\right)^{-\sigma_{XU}} = 1-\alpha_t$$

$$\frac{w_t(1)N_t(1)}{w_t(0)N_t(0)} = \frac{A_{XY}(1-A_{KX})\left(\frac{X_t}{N_t(1)}\right)^{-\sigma_{KS}}\left(\frac{Y_t}{X_t}\right)^{-\sigma_{XU}}}{(1-A_{XY})\left(\frac{Y_t}{N_t(0)}\right)^{-\sigma_{XU}}}$$

$$\frac{w_t(1)N_t(1)}{w_t(0)N_t(0)} = \frac{A_{XY}}{1-A_{XY}}(1-A_{KX})\frac{\left(\frac{X_t}{N_t(1)}\right)^{-\sigma_{KS}}}{\left(\frac{X_t}{N_t(0)}\right)^{-\sigma_{XU}}}$$

$$\begin{aligned}\frac{X_t}{N_t(1)} &= \frac{[A_{KX}K_t^{\sigma_{KS}} + (1-A_{KX})N_t(1)^{\sigma_{KS}}]^{\frac{1}{\sigma_{KS}}}}{N_t(1)} \\ &= \left[A_{KX}\left(\frac{K_t}{N_t(1)}\right)^{\sigma_{KS}} + 1 - A_{KX}\right]^{\frac{1}{\sigma_{KS}}}\end{aligned}$$

$$\frac{K_t}{Y_t} = \left[A_{KX} + (1-A_{KX})\left(\frac{K_t}{N_t(1)}\right)^{-\sigma_{KS}}\right]^{-\frac{1}{\sigma_{KS}}}\left[A_{XY} + (1-A_{XY})\left(\frac{X_t}{N_t(1)}\frac{N_t(1)}{N_t(0)}\right)^{-\sigma_{XU}}\right]^{-\frac{1}{\sigma_{XU}}}$$

The capital-output ratio is a function of the capital-skilled labor ratio:

$$\begin{aligned}\frac{K_t}{Y_t} &= \left[A_{KX} + (1-A_{KX})\left(\frac{K_t}{N_t(1)}\right)^{-\sigma_{KS}}\right]^{-\frac{1}{\sigma_{KS}}} \\ &\quad\times\left[A_{XY} + (1-A_{XY})\left(\left[A_{KX}\left(\frac{K_t}{N_t(1)}\right)^{\sigma_{KS}} + 1 - A_{KX}\right]^{\frac{1}{\sigma_{KS}}}\frac{N_t(1)}{N_t(0)}\right)^{-\sigma_{XU}}\right]^{-\frac{1}{\sigma_{XU}}} \qquad (43)\end{aligned}$$

The Bureau of Labor Statistics makes it relatively easy to access employment numbers and median income based on education level, but not mean income. However, since we are not particularly concerned with the 1% of the 1% median incomes may be sufficient. In 2012, the median annual wage was \$34750 and the median wage for a person with a professional degree was \$96420, so we set our calibration target for the education premium to

$$\frac{w(1)}{w(0)} = 2.77. \qquad (44)$$

3.00% of the work force had a doctoral or professional degree. Thus our target for the labor ratio is $\frac{N(1)}{N(0)} = 0.0300$. However, in our model they are only middle-aged workers. The rest of the population consists of both young and middle-aged workers. Thus our calibration for the ratio of the two types in the population is

$$\frac{n(1)}{n(0)} = 0.0597. \qquad (45)$$

This implies that the share of labor income going to skilled workers is

$$\begin{aligned}\frac{w(1)N(1)}{w(0)N(0)+w(1)N(1)} &= \frac{\frac{w(1)N(1)}{w(0)N(0)}}{1+\frac{w(1)N(1)}{w(0)N(0)}} \\ &= 1-\frac{1}{1+\frac{w(1)N(1)}{w(0)N(0)}} \\ &= 1-\frac{1}{1+(2.77)(0.03)} = 0.0765\end{aligned}$$

In this three-period model, it is not possible to get a positive utility cost for education with such a low skill premium with reasonable interest rates. Our calibration procedure is thus to minimize a loss function rather than to try to exactly hit our calibration targets. Another calibration target is the shape of the consumption hump. Following Gourinchas and Parker (2002), we target $\frac{C_1^*}{C_0^*} = 1.40$ and $\frac{C_2^*}{C_1^*} = 0.991$, where

$$C_{t,s} = n_0 c_{t,s}(0) + n_1 c_{t,s}(1). \tag{46}$$

Another observable that we presently do not include in the calibration, for lack of data, is the return to high education R^e_{t+1}, which satisfies

$$\frac{w_{t+1}(1)-w_{t+1}(0)}{R^e_{t+1}} - \tau - w_t(0) = 0$$

Thus the gross return to education is

$$R^e_{t+1} = \frac{w_{t+1}(1)-w_{t+1}(0)}{w_t(0)+\tau}. \tag{47}$$

The model has five technology parameters: the elasticities χ_{XU} and χ_{KS}; the shares in the CES aggregators A_{KX} and A_{XY}; and the depreciation rate δ. There are two institutional parameters related to the financing of education: the tuition τ and the allowable consumption that can be financed by borrowing c_{edu}. Finally, there are three preference parameters: the discount factor β, the inverse elasticity of intertemporal substitution γ, and the utility cost of education ρ. For the parameter vector $\theta = (\chi_{XU}, \chi_{KS}, A_{KX}, A_{XY}, \delta, \tau, c_{edu}, \beta, \gamma, \rho)$, our loss function is

$$\begin{aligned}\mathcal{L}(\theta) \;=\; &\left\{ \left(\frac{\chi_{XU}-0.8}{0.2}\right)^2 + (\chi_{KS}-4.0)^2 + \left(\frac{\frac{K}{Y}-3.0}{0.5}\right)^6 + \left(\frac{\alpha-0.33}{0.03}\right)^2 + \left(\frac{\frac{w(1)}{w(0)}-2.77}{5.0}\right)^6 \right. \\ &\left. + \left(\frac{\frac{N(1)}{N(0)}-0.03}{0.01}\right)^2 + \left(\frac{\frac{C}{Y}-0.75}{0.05}\right)^2 + \left(\frac{\frac{C_1}{C_0}-1.15}{0.05}\right)^2 + \left(\frac{\frac{C_2}{C_1}-0.74}{0.1}\right)^6 \right\} + 10^8\Theta(-\rho),\end{aligned}$$

where $\Theta(x)$ is the step function, equal to 1 for positive x and 0 for negative x. Each term consists of the ratio of the difference between the observable and its target value to a “standard error” in the estimation of that observable all raised to an even power. For $\frac{K}{Y}$, the education premium, and the retirement consumption ratio, we use a higher power of 6 rather than 2 because we really do not have good point estimates of these variables but have upper and lower bounds for what we consider reasonable values.

The baseline calibration that minimizes this loss function is given in Table 1. The depreciation rate and discount factor are reported in annual terms.

χ_{XU}	χ_{KS}	A_{KX}	A_{XY}	δ_{ann}	τ	c_{edu}	β_{ann}	γ	ρ
3.905	0.817	0.594	0.957	0.331	0.00077	0.00159	0.967	0.648	0.0245

Table 1: Baseline calibration parameters.

Variable	Model	Target
χ_{KS}	0.817	0.8
χ_{XU}	3.905	4.0
K/Y	3.385	3.0
α	0.293	0.33
$w(1)/w(0)$	6.546	2.77
r^e_{ann}	0.0871	0.1
$N(1)/N(0)$	0.0346	0.03
C/Y	0.830	0.75
C_1/C_0	1.150	1.15
C_2/C_1	0.837	0.74

Table 2: Baseline observables

The corresponding observables are in Table 2.[10] The net return on capital r^e_{ann}, which does not affect the loss function–though we get a reasonable value–is given in annual terms.

We also consider an alternate calibration in which the inverse elasticity of substitution is fixed at $\gamma = 2.5$. This has parameters reported in Table [11]. The corresponding observables are in Table 4.

4 Transition Dynamics Following a Mass Deportation

We need to compute the full model to account for the effect of changing occupational choice on wages and welfare for the two types of households. Details of how the transition path is computed are given in Appendix B. In the baseline model, deporting 5% of the low-educated labor supply reduces the capital stock by 3.9%. The immediate effect is to reduce output by 3.9% and high-educated wages by 3.6% while low-educated wages increase by 0.24%. The interest rate increases from 2.78% to 2.84%. This slight increase in the interest rate causes the initial population of low-educated old households to gain the equivalent of 0.16% of consumption and high-educated old households to gain the equivalent of 0.28%. The change in wages cause the initial population of low-educated middle-aged households to gain the equivalent of 0.13% of consumption. Note that the change in low-educated wages is so small that the higher interest rate, small as that is, has a bigger effect on the welfare of the initial retirees than the effect of their higher wages. The initial population of high-educated middle-aged households, on the other hand, because their wages fall, experience a welfare loss equivalent to 2.86% of consumption. The deportation hurts high-educated workers from this initial cohort of middle-aged households much more than it helps low-educated workers.

[10] The minimum value of the loss function is 5.569.
[11] The resulting loss is 12.08.

χ_{XU}	χ_{KS}	A_{KX}	A_{XY}	δ_{ann}	τ	c_{edu}	β_{ann}	γ	ρ
3.883	0.988	0.661	0.970	1.0	0.00000615	0.00716	0.950	2.5	0.0287

Table 3: Parameters of calibration with γ fixed at 2.5.

Variable	Model	Target
χ_{KS}	0.988	0.8
χ_{XU}	3.882	4.0
K/Y	3.000	3.0
α	0.286	0.33
$w(1)/w(0)$	8.338	2.77
r^e_{ann}	0.105	0.1
$N(1)/N(0)$	0.0299	0.03
C/Y	0.850	0.75
C_1/C_0	1.124	1.15
C_2/C_1	0.858	0.74

Table 4: Observable values for alternate calibration with $\gamma = 2.5$.

t	$\frac{\Delta Y}{Y}$	$\frac{\Delta(\frac{Y}{N})}{\frac{Y}{N}}$	$\frac{\Delta K}{K}$	$\frac{\Delta w(0)}{w(0)}$	$\frac{\Delta w(1)}{w(1)}$	$\frac{n(0)}{N}$	$\frac{n(1)}{N}$	$EV(0)$	$EV(1)$
−3	0.0	0.0	0.0	0.0	0.0	0.93534	0.06466	0.0	0.0
−2	0.0	0.0	0.0	0.0	0.0	0.93534	0.06466	0.00158	0.00283
−1	0.0	0.0	0.0	0.0	0.0	0.93534	0.06466	0.00132	−0.0286
0	−0.0393	0.00781	−0.0387	0.00243	−0.03603	0.93535	0.06465	0.00152	0.00148
1	−0.0461	0.00070	−0.0444	0.00018	0.00193	0.93532	0.06468	0.00012	0.00012
2	−0.0466	0.00021	−0.0462	0.00006	0.00016	0.93534	0.06466	0.00004	0.00004
3	−0.0467	0.00002	−0.0467	0.00001	0.00004	0.93534	0.06466	0.000004	0.000004
4	−0.0468	0.00001	−0.0468	0.000001	0.00001	0.93534	0.06466	0.000001	0.000005
5	−0.0468	0.000001	−0.0468	0.0000001	0.000001	0.93534	0.06466	0.000000	0.000000

Table 5: Transition dynamics following mass deportation for the baseline calibration.

The time series for key variables both before and after the deportation are reported in Table 5. The deportation occurs unexpectedly at $t = 0$. Note that at $t = -3$, the economy is in the preexisting steady state. $EV(s)$ at time t is the equivalent variation for type s households born at t, relative to their steady state utility.[12] Likewise, $n(s)/N$ at t is the fraction of young agents born at t who choose to become highly educated ($s = 1$) or choose not to ($s = 0$). The cohort born at $t = -2$ is the cohort of households that are old when the deportation occurs. The cohort born at $t = -1$ is the cohort of households that are middle-aged when the deportation occurs. The cohort born at $t = 0$ is young when the deportation occurs.

Let us focus on what happens to wages immediately after the deportation. Low-educated wages increase by 0.24% while high-educated wages fall by 3.6%. Why does the deportation have such a small impact on low-educated wages? From Section 2, we know that the elasticity of low-educated wages with respect to low-educated labor is the share of output going to capital and high-educated labor divided by the elasticity of substitution between low-educated labor and capital/high-educated labor. With our calibration, the elasticity of substitution is 3.9. Meanwhile the share of output going to low-educated labor is 0.58. Thus the elasticity is

$$\varepsilon = -\frac{0.58}{3.9} = -0.11.$$

If 5% of the low-skilled labor supply is removed, low-educated wages will increase by 0.55%. In the full

[12] For $\gamma \neq 1$, even though the utility of the two types of households born at any $t \notin \{-1, -2\}$ (as these two cohorts will be surprised when the deportation occurs), the equivalent variations need not be the same because of the utility cost ρ that separates the two types. See Appendix A for details of how the equivalent variation is computed.

t	$\frac{\Delta Y}{Y}$	$\frac{\Delta\left(\frac{Y}{N}\right)}{\frac{Y}{N}}$	$\frac{\Delta K}{K}$	$\frac{\Delta w(0)}{w(0)}$	$\frac{\Delta w(1)}{w(1)}$	$\frac{n(0)}{N}$	$\frac{n(1)}{N}$	$EV(0)$	$EV(1)$
−3	0.0	0.0	0.0	0.0	0.0	0.94359	0.05641	0.0	0.0
−2	0.0	0.0	0.0	0.0	0.0	0.94359	0.05641	0.00136	0.00001
−1	0.0	0.0	0.0	0.0	0.0	0.94359	0.05641	−0.00047	−0.00118
0	−0.0391	0.00848	−0.0413	0.00228	−0.03104	0.94563	0.05437	0.00106	0.00106
1	−0.0491	−0.00197	−0.0376	−0.00069	0.03309	0.94289	0.05711	−0.00036	−0.00036
2	−0.0462	0.00097	−0.0494	0.00031	−0.01036	0.94389	0.05611	0.00015	0.00015
3	−0.0475	−0.00035	−0.0460	−0.00012	0.00451	0.94348	0.05652	−0.00006	−0.00006
4	−0.0470	0.00014	−0.0476	0.00005	−0.00172	0.94364	0.05636	0.00002	0.00002
5	−0.0472	−0.00001	−0.0470	−0.00002	0.00069	0.94358	0.05642	−0.00001	−0.00001
6	−0.0472	0.00002	−0.0472	0.00001	−0.00027	0.94360	0.05640	−0.000002	−0.000002
7	−0.0472	−0.00001	−0.0472	−0.000003	0.00011	0.94359	0.05641	−0.000001	−0.000001
8	−0.0472	0.000001	−0.0472	0.000001	−0.00004	0.94359	0.05641	0.000001	0.000001
9	−0.0472	−0.000001	−0.0472	0.000000	0.00002	0.94359	0.05641	0.000000	0.000000
10	−0.0472	0.000001	−0.0472	0.000000	0.000000	0.94359	0.05641	0.000000	0.000000

Table 6: Transition dynamics following mass deportation for the alternate calibration with $\gamma = 2.5$.

model, the increase in low-educated wages is dampened further by the removal of the illegal immigrants' capital. On top of the loss of labor, the capital stock decreases by 3.9%. This reduces the capital/high-educated labor aggregator, X, by 2.7%. Since both elasticities are ultimately just functions of the factor ratio $X/N(0)$, the elasticity of low-educated labor with respect to X is the same as the elasticity with respect to low-educated labor except it has the opposite sign:

$$\frac{\partial \ln w(0)}{\partial \ln N(0)} = -\frac{\partial \ln w(0)}{\partial \ln\left(\frac{X}{N(0)}\right)} = \frac{\partial \ln w(0)}{\partial \ln X}.$$

Thus the total increase in low-educated wages is approximately $(0.11)(5\% - 2.7\%) = 0.25\%$. The elasticity of high-educated wages with respect to either low-educated labor supply or capital is more complicated, but since the elasticity of substitution between capital and high-educated labor is less than 1, we can see that the elasticity of the high-educated wage with respect to capital ought to be roughly four times as large as the elasticity of the low-educated wage with respect to capital. Moreover, while the decrease in low-educated labor and capital have countervailing effects on low-educated wages, they both work together to reduce high-educated wages. Thus the fall in high-educated wages is an order of magnitude larger than the rise in low-educated wages.

While the initial cohort of middle-aged households is hurt by the deportation, all the ensuing cohorts benefit, albeit only slightly, from the deportation. This is a consequence of the increase in per capita income that follows the deportation. The deportation reduces the total population by 4.68%, but the capital stock is only reduced by 3.87% because the immigrant households that are deported, being low-educated, hold less capital than the average household, which reflects both the low- and the high-educated households. After only a few generations though, all the intrinsic variables for the economy revert to their steady state values while extrinsic variables like output and the capital stock are reduced by 4.68% in proportion to the lost population.

However, the concentration of the welfare loss to the initial cohort of middle-aged, high-educated workers is not a general result. The time series for the alternate calibration with $\gamma = 2.5$ are reported in Table 6.

In this case there are slightly more low-educated households in the steady state, so the population, output, and the capital stock fall by 4.72% in the long run, as opposed to 4.68% in the baseline calibration. However, during the reversion to the steady state, the changes in many intrinsic variables alternate in sign. Both the low- and the high-educated in the initial cohort of middle-aged households are hurt by the deportation. The young of both types benefit while the following cohort is hurt and so on. The crucial difference between the relatively uninteresting transition dynamics of the baseline calibration and the alternate calibration is in how the deportation affects occupational choices. For the baseline calibration, Table 5 shows that the deportation has hardly any impact on the mix of low- and high-educated households. In contrast, for the alternate calibration, there is an impact. 3.6% of young households, at the time of the deportation, that would have acquired high education before the deportation respond to the 3% drop in high-educated wages and the 0.2% increase in low-educated wages by forgoing higher education.

But these wage changes are smaller than they would have been if the the young households maintained the same education choice as they would have had before the deportation. Consequently, the initial cohort of high-educated middle-aged households is not hurt as much as they would be in the baseline calibration, and the initial cohort of low-educated middle-aged households does not benefit so much from the higher wages. In the meantime, because the low-educated households save more than they would in the steady state, the capital stock at $t = 1$ is higher than it would be at $t = 0$. The interest rate falls from 3.33% to 3.20%, which is also lower than the steady-state interest rate of 3.27%. The lower interest rate means the retirement savings of the initial cohort of low-educated, middle-aged households does not generate as much retirement income as in the steady state, so on net these low-educated, middle-aged households are worse off than they would have been without the deportation.

Things are different for the initial cohort of young households. Those that forgo higher education benefit from the higher low-educated wages when they are young. Those that acquire high education benefit from the higher high-educated wages in force when they hit the labor market, which follows because a smaller fraction of this population became highly educated. Because the low-educated households earn less in middle age, however, they have less to save for retirement, so the capital stock at $t = 2$ is smaller than at $t = 1$. Thus both types earn a higher return on their retirement saving than the previous cohort, and both types are better off than they would have been without the deportation, as they must be since their utility is the same in equilibrium.

The cohort born at $t = 1$ sees that high-educated wages are 3% higher than in the steady state and low-educated wages are 0.07% lower, so slightly more of them get higher education that in the steady state. However, at $t = 2$, because there are more highly-educated households, high-educated wages are 1% lower than in the steady state and low-educated wages are 0.03% higher. An oscillation in factor prices sets in. This oscillation works against the cohorts born in odds period and works in favor of the cohorts born in even periods. Gradually, the oscillation dampens, and the economy reverts to the steady state.

Note that it is the increase in impatience rather than the decrease in the elasticity of intertemporal substitution that results in this oscillatory behavior. If we set $\gamma = 0.648$ again, as in the baseline calibration, the oscillation persists, though it dies off faster, as is shown in Table 7.[13]

5 Poverty Trap

For the calibrations of Section 4, the deportation will some people and, depending on the level of patience, may even reduce the welfare of every other generation. Nevertheless the welfare effects of the deportation are never more than the equivalent of a few percent of steady state consumption. In these examples, the macroeconomic effects of a deportation are close to a wash.[14]

[13] If we do the reverse exercise of lowering β in the baseline calibration while keeping the other parameters fixed, at that very high elasticity of intertemporal substitution more impatient households stop acquiring high education.

[14] Of course, we are not taking into account the costs of implementing the deportation.

t	$\frac{\Delta Y}{Y}$	$\frac{\Delta\left(\frac{Y}{N}\right)}{\frac{Y}{N}}$	$\frac{\Delta K}{K}$	$\frac{\Delta w(0)}{w(0)}$	$\frac{\Delta w(1)}{w(1)}$	$\frac{n(0)}{N}$	$\frac{n(1)}{N}$	$EV(0)$	$EV(1)$
−3	0.0	0.0	0.0	0.0	0.0	0.90470	0.09530	0.0	0.0
−2	0.0	0.0	0.0	0.0	0.0	0.90470	0.09530	0.00123	0.00160
−1	0.0	0.0	0.0	0.0	0.0	0.90470	0.09530	0.00110	−0.02942
0	−0.0417	0.01098	−0.0498	0.00154	−0.03610	0.90516	0.09484	0.00101	0.00098
1	−0.0467	−0.00153	−0.0486	−0.00045	0.00149	0.90462	0.09538	−0.00030	−0.00029
2	−0.0450	0.00021	−0.0448	0.00006	−0.00040	0.90472	0.09528	0.00004	0.00004
3	−0.0453	−0.00006	−0.0454	−0.00002	0.00006	0.90470	0.09530	−0.00001	−0.00001
4	−0.0452	0.00001	−0.0452	0.000002	−0.00002	0.90470	0.90470	0.000001	0.000001
5	−0.0452	−0.000002	−0.0452	−0.000001	0.000002	0.90470	0.90470	0.000000	0.000000

Table 7: Transition dynamics following mass deportation for the alternate calibration but with γ fixed to its baseline value..

χ_{XU}	χ_{KS}	A_{KX}	A_{XY}	δ_{ann}	τ	c_{edu}	β_{ann}	γ	ρ
4.000	0.800	0.567	0.971	0.080	0.000000003	0.00721	0.972	4.0	0.00844

Table 8: Parameters that give rise to multiple equilibria.

At least in this simple three-period overlapping generations model, however, there is another possible outcome that would be far more grievous. Thus far we have only considered constant steady-state equilibria, but some calibrations of the model exhibit multiple steady-state equilibria. For the parameters in Table 8, there is both a constant steady-state equilibrium and a 2-cycle equilibrium. The observables for the constant equilibrium are given in Table 9.[15] The observables for the two aggregate states of the 2-cycle equilibrium are given in Table 10. Note that the returns on education reported in Table 10 are the return earned by a young person in that state, taking into account the changing factor prices. The return is high for a young person born in an odd period because the education premium will be high in the following even period when he enters the labor force. Conversely, it is low for a young person born in an even period.because the education premium will be low when he enters the labor force.

The time series following a deportation of 5% of the low-educated population are reported in Table 11.

In many respects, the story of what happens after the deportation here is the same as in Table 6. A pattern of oscillating factor prices sets in that is favorable or unfavorable depending on the parity of a

[15] The loss function is 784.

Variable	Model	Target
χ_{KS}	0.8	0.8
χ_{XU}	4.0	4.0
K/Y	4.331	3.0
α	0.323	0.33
$w(1)/w(0)$	14.295	2.77
r^e_{ann}	0.138	0.1
$N(1)/N(0)$	0.0227	0.03
C/Y	0.824	0.75
C_1/C_0	1.395	1.15
C_2/C_1	0.988	0.74

Table 9: Observables for the constant steady-state equilibrium that arises with the parameters in Table 8

Variable	Odd Period	Even Period	Target
K/Y	2.837	6.978	3.0
α	0.372	0.200	0.33
$w(1)/w(0)$	3.654	80.147	2.77
r^e_{ann}	0.240	0.054	0.1
$N(1)/N(0)$	0.071	0.003	0.03
C/Y	0.770	0.873	0.75
C_1/C_0	0.890	10.951	1.15
C_2/C_1	1.504	0.912	0.74
r_{ann}	0.053	-0.0135	N/A

Table 10: Observables in the two states of the 2-cycle steady-state equilibrium that arises with the parameters in Table 8

t	$\frac{\Delta Y}{Y}$	$\frac{\Delta\left(\frac{Y}{N}\right)}{\frac{Y}{N}}$	$\frac{\Delta K}{K}$	$\frac{\Delta w(0)}{w(0)}$	$\frac{\Delta w(1)}{w(1)}$	$\frac{n(0)}{N}$	$\frac{n(1)}{N}$	$EV(0)$	$EV(1)$
-3	0.0	0.0	0.0	0.0	0.0	0.95659	0.04341	0.0	0.0
-2	0.0	0.0	0.0	0.0	0.0	0.95659	0.04341	0.00096	0.000001
-1	0.0	0.0	0.0	0.0	0.0	0.95659	0.04341	0.07174	0.00004
0	-0.0662	-0.01932	-0.0387	0.01052	-0.04232	0.84493	0.15507	-0.11844	-0.11844
1	0.0636	0.11702	-0.2105	0.03091	-0.73098	0.99532	0.00468	0.00015	0.00015
2	-0.3119	0.00983	0.3258	-0.07193	6.19370	0.86833	0.13167	-0.12602	-0.12602
3	-0.0479	-0.00001	-0.3817	0.00681	-0.74320	0.99347	0.00653	0.00015	0.00015
4	-0.2938	-0.25832	0.1238	-0.06550	4.12888	0.86714	0.13286	-0.12451	-0.12451
5	-0.0416	0.00651	-0.3721	0.00861	-0.74213	0.99360	0.00640	0.00015	0.00015
6	-0.2946	-0.25912	0.1370	-0.06577	4.24134	0.86717	0.13283	-0.12459	-0.12459
7	-0.0418	0.00631	-0.3724	0.00856	-0.74218	0.99359	0.00641	0.00015	0.00015
8	-0.2945	-0.25911	0.1365	-0.06577	4.23784	0.86717	0.13283	-0.12459	-0.12459
9	-0.0418	0.00631	-0.3724	0.00856	-0.74218	0.99359	0.00641	0.00015	0.00015
10	-0.2945	-0.25911	0.1365	-0.06577	4.23783	0.86717	0.13283	-0.12459	-0.12459

Table 11: Transition dynamics following mass deportation for a calibration where the deportation ends in a poverty trap.

cohort's birth period. However, there is one major difference that leads to the odd cohorts benefiting from the deportation instead of the even cohorts and vice versa for the even cohorts. Young households at the time of the deportation, instead of switching from high education to low education, switch from low education to high education. In fact, 11% of households make the switch from low to high education, which is much larger than the percentage of households that gets deported. This causes a huge increase in the ratio of high-educated workers to low-educated workers, resulting in a welfare gain equivalent to 7% of steady-state consumption for the initial cohort of middle-aged low-educated households. This big switch from low to high education is counterintuitive, though, and happens for an entirely different reason from the reverse education switches that happen in the alternate calibration with $\gamma = 2.5$. By themselves, the wage dynamics resulting from the deportation should cause people to opt out of high education, not opt in. Instead, it is the interest rate dynamics precipitated by the deportation that accounts for this strange outcome.

The deportation is a trigger for the economy to transition from the constant steady-state equilibrium to the 2-cycle steady-state equilibrium. Since one state of the 2-cycle equilibrium is far worse in terms of welfare than the constant steady-state equilibrium, this transition is a coordination failure. In the even-period cohorts too many households choose high education. They do this even though the education premium they will face in the work force is small, 3.65, and their return to education is only 5.4%. The explanation is that interest rates will be negative when these households reach retirement, so more of them want the higher income they can get from higher education in order to pay for their retirement. But the reason why interest rates are negative when they reach retirement is precisely because these households are saving so much for retirement.

The cyclic equilibrium is a poverty trap for the households born in even periods, and their welfare losses are much larger than the gains of the households born in odd periods. In this case the deportation has substantial welfare effects because the deportation catalyzes a transition to the poverty trap. Ironically, though, this is the one situation in which the common intuition that a deportation will cause a big rise in low-educated wages actually holds true but not for the reasons that people would expect.

6 Closing Remarks

One factor we do not consider in the paper is the cost of implementing a mass deportation. Nothing of this sort has happened in the Western world in recent history, so we have no data on how to calibrate this cost. Nevertheless, given the cost involved in deporting individuals on a small scale, we can imagine this cost would be large. Given that the welfare improvements enjoyed by most households following a deportation of illegal immigrants would be almost negligible, these improvements would surely change sign if we did account for the cost of deportation.

A question for future research is what happens if the information shock precedes the actual emigration. Will that pass the burden from high-educated to low-educated workers? In the existing model, middle-aged, high-educated households are hurt when the deportation shock occurs because they are not expecting the drop in their wages. But if the deportation is announced a period ahead of time, some of those households will choose work over education when they are young. This will decrease the wages of middle-aged, low-educated households after the announcement. The difficulty with this experiment is that we would have to model those who are going to be deported separately from those who will not be deported (assuming this is not random). This in turn will require us to model what kind of economy the soon-to-be deported will be emigrating to. They might, for example, have a strong incentive to save before they are deported, which would exacerbate the effects of the deportation when their capital is removed from the economy.

7 Income-Expenditure Identity

Aggregate consumption is

$$C_t = n_t(0)c_{t,0}(0) + n_{t-1}(0)c_{t,1}(0) + n_{t-2}(0)c_{t,2}(0) + n_t(1)c_{t,0}(1) + n_{t-1}(1)c_{t,1}(1) + n_{t-2}(1)c_{t,2}(1) \quad (48)$$

$$\begin{aligned} C_t &= n_t(0)\left[w_t(0) - b_{t+1,1}(0)\right] + n_{t-1}(0)\left[w_t(0) + R_t b_{t,1}(0) - b_{t+1,2}(0)\right] + n_{t-2}(0)R_t b_{t,2}(0) \\ &\quad + n_t(1)\left[-\tau - b_{t+1,1}(1)\right] + n_{t-1}(1)\left[w_t(1) + R_t b_{t,1}(1) - b_{t+1,2}(0)\right] + n_{t-2}(1)R_t b_{t,2}(1) \end{aligned}$$

$$\begin{aligned} C_t &= (n_t(0) + n_{t-1}(0))w_t(0) + n_{t-1}(1)w_t(1) - n_t(1)\tau \\ &\quad - \left[n_t(0)b_{t+1,1}(0) + n_{t-1}(0)b_{t+1,2}(0) + n_t(1)b_{t+1,1}(1) + n_{t-1}(1)b_{t+1,2}(1)\right] \\ &\quad + R_t\left[n_{t-1}(0)b_{t,1}(0) + n_{t-2}(0)b_{t,2}(0) + n_{t-1}(1)b_{t,1}(1) + n_{t-2}(1)b_{t,2}(1)\right] \end{aligned}$$

$$C_t = w_t(0)N_t(0) + w_t(1)N_t(1) - N_{t+1}(1)\tau - K_{t+1} + R_t K_t$$

$$\begin{aligned} C_t &= F_{N(0)}(K_t, N_t(0), N_t(1))N_t(0) + F_{N(1)}(K_t, N_t(0), N_t(1))N_t(1) - N_{t+1}(1)\tau - K_{t+1} \\ &\quad + \left(F_K(K_t, N_t(0), N_t(1)) + 1 - \delta\right) K_t \\ &= F_{N(0)}(K_t, N_t(0), N_t(1))N_t(0) + F_{N(1)}(K_t, N_t(0), N_t(1))N_t(1) + F_K(K_t, N_t(0), N_t(1))K_t \\ &\quad - N_{t+1}(1)\tau + (1-\delta)K_t - K_{t+1} \\ &= F(K_t, N_t(0), N_t(1)) - N_{t+1}(1)\tau + (1-\delta)K_t - K_{t+1} \\ &= Y_t - N_{t+1}(1)\tau + (1-\delta)K_t - K_{t+1} \end{aligned}$$

Investment in this model is

$$I_t = Y_t - C_t = K_{t+1} - (1-\delta)K_t + N_{t+1}(1)\tau. \quad (49)$$

A Computing Equivalent Variation

There is a small difficulty that arises when calculating the equivalent variation because of the utility cost of high education. When comparing the utility of a high-educated household to a contemporaneous low-educated household, there is no problem because the equivalent variation Δ is defined by

$$\begin{aligned} & u(c_{t,0}(0)(1+\Delta);\gamma) + \beta u(c_{t+1,1}(0)(1+\Delta);\gamma) + \beta^2 u(c_{t+2,2}(0)(1+\Delta);\gamma) \\ = \; & u(c_{t,0}(1);\gamma) + \beta u(c_{t+1,1}(1);\gamma) + \beta^2 u(c_{t+2,2}(1) - \rho \end{aligned} \quad (50)$$

If $\gamma = 1$, this simplifies to

$$(1 + \beta + \beta^2)\ln(1+\Delta) + U_t(0) = U_t(1),$$

so the usual formula

$$\Delta = \exp\left(\frac{U_t(1) - U_t(0)}{1 + \beta + \beta^2}\right) - 1$$

holds. If $\gamma \neq 1$,

$$(1+\Delta)^{1-\gamma}U_t(0) = U_t(1)$$

$$\Delta = \left(\frac{U_t(1)}{U_t(0)}\right)^{\frac{1}{1-\gamma}} - 1,$$

so again the usual formula holds.

On the other hand, if we calculate the equivalent variation based on the pre-existing steady state, for the skilled households, we have to account for ρ. Then

$$\begin{aligned} & u(c_{-3,0}(1)(1+\Delta);\gamma) + \beta u(c_{-2,1}(1)(1+\Delta);\gamma) + \beta^2 u(c_{-1,2}(1)(1+\Delta);\gamma) - \rho \\ = \quad & u(c_{t,0}(1);\gamma) + \beta u(c_{t+1,1}(1);\gamma) + \beta^2 u(c_{t+2,2}(1);\gamma) - \rho \end{aligned}$$

If $\gamma = 1$, this simplifies to

$$\begin{aligned} & (1+\beta+\beta^2)\ln(1+\Delta) + \ln c_{-3,0}(1) + \beta \ln c_{-2,1}(1) + \beta^2 \ln c_{-1,2}(1) - \rho \\ = \quad & \ln c_{t,0}(1) + \beta \ln c_{t+1,1}(1) + \beta^2 \ln c_{t+2,2}(1) - \rho, \end{aligned}$$

so

$$(1+\beta+\beta^2)\ln(1+\Delta) + U_{-3}(1) = U_t(1),$$

and we again get the usual formula

$$\Delta = \exp\left(\frac{U_t(1) - U_{-3}(1)}{1+\beta+\beta^2}\right) - 1.$$

However, if $\gamma \neq 1$, it simplifies to

$$\begin{aligned} & (1+\Delta)^{1-\gamma} \frac{c_{-3,0}^{1-\gamma}(1) + \beta c_{-2,1}^{1-\gamma}(1) + \beta^2 c_{-1,2}^{1-\gamma}(1)}{1-\gamma} - \rho \\ = \quad & \frac{c_{t,0}^{1-\gamma}(1)(1+\Delta);\gamma) + \beta c_{t+1,1}^{1-\gamma}(1) + \beta^2 c_{t+2,2}^{1-\gamma}(1)}{1-\gamma} - \rho \\ & (1+\Delta)^{1-\gamma}\left(U_{-3}(1) + \rho\right) - \rho = U_t(1) \end{aligned}$$

Thus we have to modify the formula to

$$\Delta = \left(\frac{U_t(1) + \rho}{U_{-3}(1) + \rho}\right)^{\frac{1}{1-\gamma}} - 1 \tag{51}$$

B Computational Procedure

We can use the iteration

$$n_t^{(i+1)}(1) = \frac{n_t^{(i)}(1)\exp(\Delta)}{1 + n_t^{(i)}(1)(\exp(\Delta) - 1)}$$

where Δ is the equivalent variation.

$$\lim_{\Delta\to\infty} n_t^{(i+1)}(1) = \frac{n_t^{(i)}(1)\exp(\Delta)}{n_t^{(i)}(1)\exp(\Delta)} = 1$$

$$\lim_{\Delta\to-\infty} n_t^{(i+1)}(1) = \lim_{\Delta\to-\infty} \frac{n_t^{(i)}(1)}{1 - n_t^{(i)}(1)}\exp(\Delta) = 0$$

For small Δ,

$$n_t^{(i+1)}(1) = n_t^{(i)}(1)\frac{1+\Delta}{1+n_t^{(i)}(1)\Delta} + O(\Delta^2) = n_t^{(i)}(1)(1+(1-n_t^{(i)}(1))\Delta) + O(\Delta^2).$$

$$\begin{aligned}
\frac{\partial n_t^{(i+1)}(1)}{\partial \Delta} &= \frac{n_t^{(i)}(1)\exp(\Delta)}{1+n_t^{(i)}(1)(\exp(\Delta)-1)} - \frac{(n_t^{(i)}(1)\exp(\Delta))^2}{[1+n_t^{(i)}(1)(\exp(\Delta)-1)]^2} \\
&= \frac{n_t^{(i)}(1)\exp(\Delta)[1+n_t^{(i)}(1)(\exp(\Delta)-1)] - (n_t^{(i)}(1)\exp(\Delta))^2}{[1+n_t^{(i)}(1)(\exp(\Delta)-1)]^2} \\
&= \frac{\exp(\Delta)(1-n_t^{(i)}(1))n_t^{(i)}(1)}{[1+n_t^{(i)}(1)(\exp(\Delta)-1)]^2} \geq 0
\end{aligned}$$

for $n_t^{(i)}(1) \in [0,1]$.

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