Taxing Capitalists*

Preliminary and Incomplete

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Abstract

With rising wealth inequality, there is growing interest in the question of how to tax the very rich. A gaping difference between overlapping-generations model and infinite-horizon models is the result that it is generally optimal to tax capital income in the former while it is generally optimal not to tax capital at all in the latter. This last result extends to models where some fraction of the population exhibit pure altruism toward their descendants. In particular, it applies to a segregated economy model with Cobb-Douglas production and price-taking capitalists. Here we show that, unlike a capital tax, a progressive consumption tax that only applies to capitalists is nondistortionary. Tremendous welfare gains are possible for the whole population if taxes on capital are replaced by such a consumption tax. The political economy that supports the tax structure of most countries today is thus rather puzzling. However, these welfare gains do come at the cost of greater wealth inequality.

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Rising wealth inequality has provoked considerable discussion about policies that might counteract this trend. In his book documenting the problem, Piketty (2014) proposed a progressive wealth tax as the solution, and this idea has been pushed by many politicians on the left, especially Elizabeth Warren. However, there is a huge bifurcation in the literature on taxing capital. With the infinite-horizon models that used to be the norm in macroeconomics, there is a fairly robust finding that capital taxes should be set to zero (Judd (1985)) and Chamley (1986)). In overlapping-generations models, results are more

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mixed (Conesa, Kitao, and Krueger (2009)). In the extreme case of a two-period overlapping-generations (OLG) model, it is the tax on labor income that should usually be set to zero unless that would require a prohibitively high tax on capital.

The segregated-economy model of Feigenbaum (2018) is a hybrid of these two classes of models. Since the bulk of the population, the "laborers", do not present evidence of a bequest motive (Hurd (1987)), presumably because they do not accumulate enough assets to pass onto the next generation, we model them as finite-lived households that live for two periods. Inequality results from the existence of a handful of households, the "capitalists", wealthy enough to behave as infinite-lived dynasties à la Barro (1974).[1] We show here that the Judd-Chamley result that capital taxation is Pareto inefficient still holds in the segregated-economy model, assuming Cobb-Douglas production and price-taking capitalists.[2] Wealth taxes do reduce wealth inequality, but they are also highly distortionary.

Following a proposal of Raei (2018) to have the Internal Revenue Service collect consumption taxes by giving a credit for any saving, we modify the segregated economy model of Feigenbaum (2018) to allow for consumption taxes that discriminate between the two types. Unlike a tax on capital income, a constant tax on the consumption of capitalists is nondistortionary. Laborers would obviously prefer to tax the consumption of capitalists as opposed to labor income while capitalists would prefer the opposite. We can then solve for the optimal political economy as a bargaining problem between the two types. The absence of a significant tax on the consumption of capitalists implies that capitalists have considerably more bargaining power than laborers. However, no bargaining solution can explain why we have a tax on capital income–if capitalists are, indeed, price takers.

The paper is organized as follows. We describe the model in Section 1. Our main results that a tax on the consumption of capitalists is nondistortionary and that laborers would prefer not to tax capital income if production is Cobb-Douglas is derived in Section 2. The laborer's problem is solved for the special case of log utility in Section 3. The price-taking capitalist's problem is solved, also for the case of log utility, in Section 4. With these solutions, we can say more about what capitalists think about taxing capital in Section 5 and about the optimal tradeoff between taxing labor and capitalist's consumption in Section 6. Quantitative results are provided in Section 7. We conclude in Section 111.

1 The Model

[1] Feigenbaum and Li (2018) provide microfoundations for the coexistence of these two models in a setting where parents and children bargain over transfers between them.

[2] The segregated-economy model also allows for the possibility that capitalists are price-setters. We will discuss this case further in the Conclusion.

We augment the original segregated economy model of Feigenbaum (2018) with consumption taxes that depend on type: capitalist (c) or laborer (l). Time is discrete. In each period a measure μ of laborers is born while the measure of capitalists is $1-\mu$. Laborers live for two periods while capitalists are infinitely-lived, so the total population is $1+\mu$.

A laborer born at t chooses his consumptions $\{c^l_{t,0}, c^l_{t+1,1}\}$ and leisure l_t to maximize his lifetime utility

$$U^l_t = \max u^l(c^l_{t,0}, l_t) + \beta_l u^l(c^l_{t+1,1}, 1) \tag{1}$$

subject to

$$(1+\tau^{c,l}_t)c^l_{t,0} + k^l_{t+1} \leq (1-\tau^l_t)w_t(1-l_t) \tag{2}$$

$$(1+\tau^{c,l}_{t+1})c^l_{t+1,1} \leq (1+(1-\tau^k_{t+1})r_{t+1})k^l_{t+1}, \tag{3}$$

$$c^l_{t,0}, c^l_{t+1,1}, l_t \geq 0, \tag{4}$$

and

$$l_t \leq 1, \tag{5}$$

where k^l_{t+1} is the saving of a laborer at t. The government imposes taxes τ^l_t on labor income, τ^k_t on capital income, and $\tau^{c,l}_t$ on the consumption of laborers. The wage w_t and the before-tax return on capital r_t will be treated as given by the laborer.

Meanwhile, the capitalists live forever. They should be viewed as an infinitely-lived dynasty, and their population at t is the current generation of decision-making patriarchs. Given the dynasty's current capital k_t,[3] a capitalist at t maximizes

$$U^c_t = \sum_{s=0}^{\infty} \beta^s_c u^c(c^c_{t+s}) \tag{6}$$

subject to

$$(1+\tau^{c,c}_t)c^c_t + k_{t+1} = (1+(1-\tau^k_t)r_t)k_t \tag{7}$$

and the no-Ponzi condition

$$\lim_{s\to\infty} \frac{k_{t+s+1}}{\prod_{i=0}^{s}(1+(1-\tau^k_{t+i})r_{t+i})} \geq 0. \tag{8}$$

Note that consumption may be taxed at a separate rate $\tau^{c,c}_t$ from the rate $\tau^{c,l}_t$ that it is taxed at for laborers.

Let D_t constitute government debt. Then the capital stock is

$$K_t = \mu k^l_t + (1-\mu)k_t - D_t \tag{9}$$

and the labor supply is

[3] Since k^c_t will serve as a state variable and appears far more often than its counterpart k^l_t, we suppress the superscript c.

$$N_t = \mu(1 - l_t). \tag{10}$$

There is a constant returns to scale production function $F(K, N)$ such that

$$w_t = w(K_t, N_t) = F_N(K_t, N_t) \tag{11}$$

and

$$r_t = r(K_t, N_t) = F_K(K_t, N_t) - \delta, \tag{12}$$

where $\delta > 0$ is the depreciation rate. It is helpful to denote the after-tax gross return on capital as

$$R_t = R_t(K_t, N_t) = 1 + (1 - \tau_t^k) r(K_t, N_t). \tag{13}$$

The taxes, and possibly debt, are used to finance government expenditures of goods G_t. The government must satisfy its budget constraint

$$G_t + (1+r_t)D_t = \tau_t^l w_t N_t + r_t \tau_t^k K_t + \mu \tau_t^{c,l}(c_{t,0}^l + c_{t,1}^l) + (1-\mu)\tau_t^{c,c} c_t^c + D_{t+1}. \tag{14}$$

Aggregate consumption is

$$C_t = \mu(c_{t,0}^l + c_{t,1}^l) + (1 - \mu)c_t^c \tag{15}$$

$$C_t^l = \mu(c_{t,0}^l + c_{t,1}^l) = \frac{\mu}{1 + \tau_t^{c,l}} \left[(1 - \tau_t^l) w_t (1 - l_t) - k_{t+1}^l + (1 + (1 - \tau_t^k) r_t) k_t^l\right] \tag{16}$$

$$(1 + \tau_t^{c,l}) C_t^l = w_t N_t - \tau_t^l w_t N_t + \mu \left[-k_{t+1}^l + (1 + r_t) k_t^l\right] - \mu \tau_t^k r_t k_t^l$$

$$(1 + \tau_t^{c,l}) C_t^l = w_t N_t - \tau_t^l w_t N_t + \mu \left[-k_{t+1}^l + (1 + r_t) k_t^l\right] - \mu \tau_t^k r_t k_t^l$$

$$C_t^c = (1 - \mu) c_t^c = \frac{1 - \mu}{1 + \tau_t^{c,c}} \left[(1 + (1 - \tau_t^k) r_t) k_t - k_{t+1}\right] \tag{17}$$

$$(1 + \tau_t^{c,c}) C_t^c = (1 - \mu) \left[(1 + r_t) k_t - k_{t+1}\right] - (1 - \mu) \tau_t^k r_t k_t$$

$$\begin{aligned} C_t + \tau_t^{c,l} C_t^l + \tau_t^{c,c} C_t^c \quad &= \quad w_t N_t - \tau_t^l w_t N_t + \mu \left[-k_{t+1}^l + (1 + r_t) k_t^l\right] - \mu \tau_t^k r_t k_t^l \\ &\quad + (1 - \mu) \left[(1 + r_t) k_t - k_{t+1}\right] - (1 - \mu) \tau_t^k r_t k_t \end{aligned}$$

$$\begin{aligned} C_t \quad &= \quad w_t N_t + (1 + r_t) \left[\mu k_t^l + (1 - \mu) k_t\right] - \mu k_{t+1}^l - (1 - \mu) k_{t+1} \\ &\quad - \tau_t^l w_t N_t - \tau_t^k r_t K_t - \tau_t^{c,l} C_t^l - \tau_t^{c,c} C_t^c \end{aligned}$$

From (14),

$$\begin{aligned} C_t \quad &= \quad w_t N_t + (1 + r_t) \left[\mu k_t^l + (1 - \mu) k_t\right] - \mu k_{t+1}^l - (1 - \mu) k_{t+1} \\ &\quad - G_t - (1 + r_t) D_t + D_{t+1} \end{aligned}$$

From (9),

$$C_t = w_t N_t + (1 + r_t) K_t - K_{t+1} - G_t.$$

Define investment as

$$I_t = K_{t+1} - (1-\delta)K_t. \tag{18}$$

$$\begin{aligned} C_t &= w_t N_t + (1+r_t)K_t - I_t - (1-\delta)K_t - G_t \\ &= w_t N_t + (r_t+\delta)K_t - I_t - G_t \end{aligned}$$

Thus we have the income-expenditure identity:

$$C_t + I_t + G_t = F_N(K_t, N_t)N_t + F_K(K_t, N_t)K_t = F(K_t, N_t) = Y_t. \tag{19}$$

Define

$$\kappa_t = \frac{(1-\mu)k_t}{K_t}, \tag{20}$$

which is our measure of wealth inequality.

Let us also define

$$w_t^{at} = (1-\tau_t^l)w_t, \tag{21}$$

$$R_t = 1 + (1-\tau_t^k)r_t. \tag{22}$$

and

$$p_t^i = 1 + \tau_t^{c,i} \tag{23}$$

for $i = c, l$.

2 Some Results about Capital Taxes

The Bellman equation for a price-taking capitalist[4] is

$$v_t(k_t) = u^c(c_t^c) + \beta^c v_{t+1}(k_{t+1}) \tag{24}$$

subject to

$$p_t^c c_t^c + k_{t+1} = R_t k_t. \tag{25}$$

The Lagrangian is

$$L_t^c = u^c(c_t^c) + \beta^c v_{t+1}(k_{t+1}) + \lambda_t[R_t k_t - p_t^c c_t^c - k_{t+1}]. \tag{26}$$

The first-order conditions are

$$\frac{\partial L_t^c}{\partial c_t^c} = (u^c)'(c_t^c) - \lambda_t p_t^c = 0. \tag{27}$$

$$\frac{\partial L_t^c}{\partial k_{t+1}} = \beta^c v'_{t+1}(k_{t+1}) - \lambda_t = 0. \tag{28}$$

[4] See Feigenbaum (2018) for the case of a price-setting capitalist.

Thus

$$(u^c)'(c_t^c) = p_t^c \beta^c v_{t+1}'(k_{t+1}).$$

From the envelope theorem,

$$v_t'(k_t) = \frac{\partial L_t^c}{\partial k_t} = \lambda_t R_t = \frac{R_t}{p_t^c}(u^c)'(c_t^c).$$

Thus we get the Euler equation

$$(u^c)'(c_t^c) = \frac{p_t^c}{p_{t+1}^c} \beta^c R_{t+1} (u^c)'(c_{t+1}^c). \tag{29}$$

Thus in a steady state with constant consumption taxes, we will have the familiar condition

$$\beta^c R = 1. \tag{30}$$

Notice also that we can write the laborer's problem as

$$U_t^l = \max U^l(c_{t,0}^l, c_{t+1,1}^l, l_t)$$

subject to

$$p_t^c c_{t,0}^l + \frac{p_{t+1}^c c_{t+1,1}^l}{R_{t+1}} \leq w_t^{at}(1 - l_t), \tag{31}$$

$$c_{t,0}^l, c_{t+1,1}^l, l_t \geq 0, \tag{32}$$

and

$$l_t \leq 1. \tag{33}$$

In the steady state, the gross after-tax return on capital is fixed by (30). If we also hold the consumption tax fixed, the choice set defined by (31)-(33) expands if we increase w^{at}. Thus the laborer's welfare in the segregated economy model (with price-taking capitalists) is monotonically increasing in w^{at}.

Now we specialize to the case of Cobb-Douglas production:

$$F(K, N) = K^\alpha N^{1-\alpha}. \tag{34}$$

Thus the factor prices are

$$w_t = (1 - \alpha) \left(\frac{K_t}{N_t} \right)^\alpha \tag{35}$$

and

$$r_t = \alpha \left(\frac{K_t}{N_t} \right)^{\alpha - 1} - 1. \tag{36}$$

Suppose that we have only capital and labor taxes. Let us parameterize feasible equilibria by τ_k, so

$$\tau_n(\tau_k) w(K(\tau_k)) N + \tau_k r(K(\tau_k)) K(\tau_k) = G \tag{37}$$

where

$$1 + (1-\tau_k) r(K(\tau_k)) = (\beta^c)^{-1}.$$

Thus the capital stock varies with the capital tax rate as

$$K(\tau_k) = r^{-1}\left(\frac{(\beta^c)^{-1} - 1}{1-\tau_k}\right). \tag{38}$$

Then

$$\tau_n(\tau_k) = \frac{G - \tau_k r(K(\tau_k))K(\tau_k)}{w(K(\tau_k))N}$$

$$\tau_n(\tau_k) = \frac{G - \frac{\tau_k}{1-\tau_k}((\beta^c)^{-1} - 1)K(\tau_k)}{w(K(\tau_k))N}. \tag{39}$$

Note that this formula depends only on G/N and β^c along with the parameters of the production function.

For the special case of a Cobb-Douglas production function,

$$w = (1-\alpha)\left(\frac{K}{N}\right)^{\alpha}$$

$$\frac{K}{wN} = \frac{\frac{K}{N}}{(1-\alpha)\left(\frac{K}{N}\right)^{\alpha}} = \frac{\left(\frac{K}{N}\right)^{1-\alpha}}{1-\alpha}$$

$$r = \alpha\left(\frac{K}{N}\right)^{\alpha-1} - \delta$$

$$\frac{K}{N} = \left(\frac{r+\delta}{\alpha}\right)^{\frac{1}{\alpha-1}}$$

$$w = (1-\alpha)\left(\frac{r+\delta}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}$$

$$w(\tau_k) = (1-\alpha)\left(\frac{\frac{(\beta^c)^{-1}-1}{1-\tau_k} + \delta}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} \tag{40}$$

$$\frac{K}{wN} = \frac{\frac{\alpha}{r+\delta}}{1-\alpha} = \frac{\alpha}{1-\alpha}\frac{1}{r+\delta}$$

$$\tau_n(\tau_k) = \frac{1}{1-\alpha}\left(\frac{\frac{(\beta^c)^{-1}-1}{1-\tau_k} + \delta}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} \frac{G}{N} - \frac{\tau_k}{1-\tau_k}((\beta^c)^{-1} - 1)\frac{\alpha}{1-\alpha}\frac{1}{\frac{(\beta^c)^{-1}-1}{1-\tau_k} + \delta}$$

$$\tau_n(\tau_k) = \frac{1}{1-\alpha}\left[\left(\frac{\frac{(\beta^c)^{-1}-1}{1-\tau_k} + \delta}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} \frac{G}{N} - \alpha\tau_k \frac{(\beta^c)^{-1} - 1}{(\beta^c)^{-1} - 1 + \delta(1-\tau_k)}\right] \tag{41}$$

As a side note, τ_n is an increasing function of α and a decreasing function of β^c. If α is high enough or β^c is low enough, τ_n will be greater than 1.

If $\delta = 1$,

$$\tau_n(\tau_k) = \frac{1}{1-\alpha}\left[\left(\frac{\frac{(\beta^c)^{-1}-1}{1-\tau_k}+\delta}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}\frac{G}{N} - \alpha\tau_k\frac{(\beta^c)^{-1}-1}{(\beta^c)^{-1}-\tau_k}\right]. \tag{42}$$

$$w^{at}(\tau_k) = (1-\tau_n(\tau_k))w(\tau_k) = w(\tau_k) - \frac{\left[G - \frac{\tau_k}{1-\tau_k}((\beta^c)^{-1}-1)K(\tau_k)\right]}{N}$$

$$w^{at}(\tau_k) = w(\tau_k) + \frac{\tau_k}{1-\tau_k}\left((\beta^c)^{-1}-1\right)\frac{K(\tau_k)}{N} - \frac{G}{N}$$

While the first term is decreasing in τ_k, the second term appears to be increasing.

$$\frac{K(\tau_k)}{N} = \left(\frac{\frac{(\beta^c)^{-1}-1}{1-\tau_k}+\delta}{\alpha}\right)^{\frac{1}{\alpha-1}}$$

$$\frac{d}{d\tau_k}\left(\frac{K(\tau_k)}{N}\right) = \frac{1}{\alpha-1}\left(\frac{\frac{(\beta^c)^{-1}-1}{1-\tau_k}+\delta}{\alpha}\right)^{\frac{1}{\alpha-1}-1}\frac{(\beta^c)^{-1}-1}{\alpha}\frac{1}{(1-\tau_k)^2} < 0$$

$$\begin{aligned}\frac{d}{d\tau_k}\ln\left(\frac{K(\tau_k)}{N}\right) &= \frac{1}{\alpha-1}\frac{\alpha}{\frac{(\beta^c)^{-1}-1}{1-\tau_k}+\delta}\frac{(\beta^c)^{-1}-1}{\alpha}\frac{1}{(1-\tau_k)^2} \\ &= \frac{1}{\alpha-1}\frac{(\beta^c)^{-1}-1}{\frac{(\beta^c)^{-1}-1}{1-\tau_k}+\delta}\frac{1}{(1-\tau_k)^2}\end{aligned}$$

$$\frac{K'(\tau_k)}{N} = \frac{1}{\alpha-1}\frac{(\beta^c)^{-1}-1}{(\beta^c)^{-1}-1+\delta(1-\tau_k)}\frac{1}{1-\tau_k}\frac{K(\tau_k)}{N} \tag{43}$$

$$\frac{d}{d\ln\tau_k}\ln\left(\frac{K(\tau_k)}{N}\right) = \frac{1}{\alpha-1}\frac{(\beta^c)^{-1}-1}{(\beta^c)^{-1}-1+\delta(1-\tau_k)}\frac{\tau_k}{1-\tau_k} < 0 \tag{44}$$

$$w^{at}(\tau_k) = w(\tau_k) + \frac{\tau_k}{1-\tau_k}\left((\beta^c)^{-1}-1\right)\left(\frac{\frac{(\beta^c)^{-1}-1}{1-\tau_k}+\delta}{\alpha}\right)^{\frac{1}{\alpha-1}} - \frac{G}{N}$$

$$w^{at}(\tau_k) = (1-\alpha)\left(\frac{\frac{(\beta^c)^{-1}-1}{1-\tau_k}+\delta}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \frac{\tau_k}{1-\tau_k}\left((\beta^c)^{-1}-1\right)\left(\frac{\frac{(\beta^c)^{-1}-1}{1-\tau_k}+\delta}{\alpha}\right)^{\frac{1}{\alpha-1}} - \frac{G}{N}$$

$$w^{at}(\tau_k) = \left(\frac{\frac{(\beta^c)^{-1}-1}{1-\tau_k}+\delta}{\alpha}\right)^{\frac{1}{\alpha-1}}\left[\frac{1-\alpha}{\alpha}\left(\frac{(\beta^c)^{-1}-1}{1-\tau_k}+\delta\right) + \frac{\tau_k}{1-\tau_k}\left((\beta^c)^{-1}-1\right)\right] - \frac{G}{N}$$

$$w^{at}(\tau_k) = \left(\frac{\frac{(\beta^c)^{-1}-1}{1-\tau_k}+\delta}{\alpha}\right)^{\frac{1}{\alpha-1}} \left[\left(\frac{1-\alpha}{\alpha}+\tau_k\right)\frac{(\beta^c)^{-1}-1}{1-\tau_k}+\delta\frac{1-\alpha}{\alpha}\right]-\frac{G}{N}$$

$$w^{at}(\tau_k) = \frac{K(\tau_k)}{N}\left[\left(\frac{1-\alpha}{\alpha}+\tau_k\right)\frac{(\beta^c)^{-1}-1}{1-\tau_k}+\delta\frac{1-\alpha}{\alpha}\right]-\frac{G}{N}$$

$$J(\tau_k) = \left(\frac{1-\alpha}{\alpha}+\tau_k\right)\frac{(\beta^c)^{-1}-1}{1-\tau_k}+\delta\frac{1-\alpha}{\alpha} \tag{45}$$

$$\frac{\tau_k}{1-\tau_k} = \frac{1}{1-\tau_k}-1$$

$$J(\tau_k) = \frac{1-\alpha}{\alpha}\frac{(\beta^c)^{-1}-1}{1-\tau_k}+((\beta^c)^{-1}-1)\left[\frac{1}{1-\tau_k}-1\right]+\delta\frac{1-\alpha}{\alpha}$$

$$J(\tau_k) = \frac{1}{\alpha}\frac{(\beta^c)^{-1}-1}{1-\tau_k}+\delta\frac{1-\alpha}{\alpha}+1-(\beta^c)^{-1} \tag{46}$$

$$J'(\tau_k) = \frac{1}{\alpha}\frac{(\beta^c)^{-1}-1}{(1-\tau_k)^2} \tag{47}$$

$$w^{at}(\tau_k) = \frac{J(\tau_k)K(\tau_k)}{N}-\frac{G}{N}$$

$$\begin{aligned}
\frac{dw^{at}(\tau_k)}{d\tau_k} &= J'(\tau_k)\frac{K(\tau_k)}{N}+J(\tau_k)\frac{K'(\tau_k)}{N} \\
&= \left[J'(\tau_k)+J(\tau_k)\frac{1}{\alpha-1}\frac{(\beta^c)^{-1}-1}{(\beta^c)^{-1}-1+\delta(1-\tau_k)}\frac{1}{1-\tau_k}\right]\frac{K(\tau_k)}{N}
\end{aligned}$$

$$\begin{aligned}
\frac{N}{K(\tau_k)}\frac{dw^{at}(\tau_k)}{d\tau_k} &= \frac{1}{\alpha}\frac{(\beta^c)^{-1}-1}{(1-\tau_k)^2}+\left(\frac{1}{\alpha}\frac{(\beta^c)^{-1}-1}{1-\tau_k}+\delta\frac{1-\alpha}{\alpha}+1-(\beta^c)^{-1}\right) \\
&\quad \times\frac{1}{\alpha-1}\frac{(\beta^c)^{-1}-1}{(\beta^c)^{-1}-1+\delta(1-\tau_k)}\frac{1}{1-\tau_k}
\end{aligned}$$

$$\begin{aligned}
&(1-\tau_k)^2\left[(\beta^c)^{-1}-1+\delta(1-\tau_k)\right]\frac{N}{K(\tau_k)}\frac{dw^{at}(\tau_k)}{d\tau_k} \\
= &\ \frac{1}{\alpha}((\beta^c)^{-1}-1)\left[(\beta^c)^{-1}-1+\delta(1-\tau_k)\right]+\frac{((\beta^c)^{-1}-1)^2}{\alpha(\alpha-1)} \\
&+\frac{(\beta^c)^{-1}-1}{\alpha-1}\left(\delta\frac{1-\alpha}{\alpha}+1-(\beta^c)^{-1}\right)(1-\tau_k) \\
= &\ \frac{((\beta^c)^{-1}-1)^2}{\alpha}\left[1-\frac{1}{1-\alpha}\right]+\left[\frac{\delta}{\alpha}((\beta^c)^{-1}-1)-\frac{\delta}{\alpha}((\beta^c)^{-1}-1)-\frac{((\beta^c)^{-1}-1)^2}{\alpha-1}\right](1-\tau_k) \\
= &\ -\frac{((\beta^c)^{-1}-1)^2}{\alpha}\frac{\alpha}{1-\alpha}+\frac{((\beta^c)^{-1}-1)^2}{1-\alpha}(1-\tau_k) \\
= &\ -\frac{((\beta^c)^{-1}-1)^2}{1-\alpha}\tau_k
\end{aligned}$$

Thus, somewhat counterintuitively, the after-tax wage decreases with the capital tax.

$$\frac{dw^{at}(\tau_k)}{d\tau_k} = -\frac{((\beta^c)^{-1}-1)^2}{1-\alpha}\frac{\tau_k}{(1-\tau_k)^2\left[(\beta^c)^{-1}-1+\delta(1-\tau_k)\right]}\frac{K(\tau_k)}{N} < 0. \tag{48}$$

3 Laborers with Log Utility

Suppose that the laborer's period utility function is

$$u^l(c^l,l) = \begin{cases} \eta \ln c^l + (1-\eta)\ln l & \gamma^l = 1 \\ \frac{1}{1-\gamma^l}\left((c^l)^\eta l^{1-\eta}\right)^{1-\gamma^l} & \gamma^l \neq 1 \end{cases} \tag{49}$$

for $\gamma^l \geq 0$ and

$$u^c(c^c) = \begin{cases} \ln(c^c) & \gamma^c = 1 \\ \frac{1}{1-\gamma^c}(c^c)^{1-\gamma^c} & \gamma^c \neq 1 \end{cases} \tag{50}$$

for $\gamma^c \geq 0$.

For simplicity, let us specialize to the case were $\gamma^l = 1$.[5] A laborer born at time t will maximize

$$U_t^l = \eta \ln c_{t,0}^l + (1-\eta)\ln l_t + \beta^l \eta \ln c_{t+1,1}^l \tag{51}$$

subject to

$$p_t^l c_{t,0}^l + k_{t+1}^l = w_t^{at}(1-l_t) \tag{52}$$

$$p_{t+1}^l c_{t+1,1}^l = R_t^l k_{t+1}^i. \tag{53}$$

The lifetime budget constraint is

$$p_t^l c_{t,0}^l + \frac{p_{t+1}^l c_{t+1,1}^l}{R_t^l} = w_t^{at}(1-l_t). \tag{54}$$

The laborer's Lagrangian is

$$L_t^l = \eta \ln c_{t,0}^l + (1-\eta)\ln l_t + \beta^l \eta \ln c_{t+1,1}^l + \lambda_t \left[w_t^{at}(1-l_t) - p_t^l c_{t,0}^l - \frac{p_{t+1}^l c_{t+1,1}^l}{R_t^l} \right]. \tag{55}$$

$$\frac{\partial L_t^l}{\partial c_{t,0}^l} = \frac{\eta}{c_{t,0}^l} - \lambda_t p_t^l = 0 \tag{56}$$

$$\frac{\partial L_t^l}{\partial c_{t+1,1}^l} = \frac{\beta^l \eta}{c_{t+1,1}^l} - \frac{\lambda_t p_{t+1}^l}{R_t^l} = 0 \tag{57}$$

[5] We consider the more general case in Appendix 9.

$$\frac{\partial L_t^l}{\partial l_t} = \frac{1-\eta}{l_t} - \lambda_t w_t^{at} = 0 \tag{58}$$

$$c_{t,0}^l = \frac{\eta}{\lambda_t p_t^l}$$

$$c_{t+1,1}^l = \beta^l R_t^l \frac{\eta}{\lambda_t p_{t+1}^l} = \beta^l R_t^l \frac{p_t^l}{p_{t+1}^l} \frac{\eta}{\lambda_t p_t^l}$$

$$\frac{\eta}{1-\eta} \frac{l_t}{c_{t,0}^l} = \frac{p_t^l}{w_t^{at}}$$

$$l_t = \frac{1-\eta}{\eta} \frac{p_t^l c_{t,0}^l}{w_t^{at}}$$

The Euler equation is

$$c_{t+1,1}^l = \beta^l R_t^l \frac{p_t^l}{p_{t+1}^l} c_{t,0}^l. \tag{59}$$

$$l_t = \frac{1-\eta}{\lambda_t w_t^{at}}.$$

Substituting these into the budget constraint,

$$\frac{\eta}{\lambda_t} + \beta^l \frac{\eta}{\lambda_t} = w_t^{at} \left(1 - \frac{1-\eta}{\lambda_t w_t^{at}}\right) = w_t^{at} - \frac{1-\eta}{\lambda_t}$$

$$\frac{1-\eta+\eta+\eta\beta^l}{\lambda_t} = w_t^{at}$$

$$\frac{1}{\lambda_t} = \frac{w_t^{at}}{1+\eta\beta^l}$$

$$c_{t,0}^l = \frac{\eta}{1+\eta\beta^l} \frac{w_t^{at}}{p_t^l} \tag{60}$$

$$c_{t+1,1}^l = \frac{\eta\beta^l R_t^l}{1+\eta\beta^l} \frac{w_t^{at}}{p_{t+1}^l} \tag{61}$$

$$l_t = \frac{1-\eta}{w_t^{at}} \frac{w_t^{at}}{1+\eta\beta^l}$$

$$l_t = \frac{1-\eta}{1+\eta\beta^l} \tag{62}$$

$$1 - l_t = \frac{1+\eta\beta^l - 1 + \eta}{1+\eta\beta^l} = \frac{\eta+\eta\beta^l}{1+\eta\beta^l} \in [0,1] \tag{63}$$

$$N_t = N = \eta\mu \frac{1+\beta^l}{1+\eta\beta^l}. \tag{64}$$

Since the labor supply is constant, we can define

$$w_t = w(K_t) = (1-\alpha)\left(\frac{K_t}{N}\right)^{\alpha} \tag{65}$$

$$w_t^{at} = w^{at}(K_t) = (1-\tau_t^n)w(K_t) \tag{66}$$

$$R_t^i = R^i(K_t) = 1 + (1-\tau_t^{i,l})\left[\alpha\left(\frac{K_t}{N}\right)^{\alpha-1} - \delta\right] \tag{67}$$

$$\begin{aligned} k_{t+1}^l &= w_t^{at}(1-l_t) - p_t^l c_{t,0}^l \\ &= w_t^{at}\frac{\eta+\eta\beta^l}{1+\eta\beta^l} - \frac{\eta}{1+\eta\beta^l}w_t^{at} \end{aligned}$$

$$k_{t+1}^l = \frac{\eta\beta^l}{1+\eta\beta^l}w_t^{at}. \tag{68}$$

We an define the saving rate as

$$s = \frac{\eta\beta^l}{1+\eta\beta^l}. \tag{69}$$

Note that a constant consumption tax on laborers would not affect the Euler equation (59). However, a constant consumption tax on laborers would still be distortionary since, if $\tau_t^n = \tau_t^k = 0$ for all t, the marginal rate of substitution between young consumption and leisure will be

$$\frac{\frac{\partial U_t}{\partial c_{t,0}^l}}{\frac{\partial U_t}{\partial l_t}} = \frac{\frac{\eta}{c_{t,0}^l}}{\frac{1-\eta}{l_t}} = \frac{\eta}{\frac{\eta}{1+\eta\beta^l}\frac{w_t^{at}}{p_t^l}}\frac{\frac{1-\eta}{1+\eta\beta^l}}{1-\eta} = \frac{p_t^l}{w_t^{at}} = \frac{1+\tau_t^{c,l}}{w_t} \tag{70}$$

when it should be w_t^{-1} in a Pareto efficient allocation.

4 A Price-Taking Capitalist with Log Utility

A capitalist solves the Bellman equation

$$v_t(k_t) = \ln(c_t^c) + \beta^c v_{t+1}(k_{t+1}) \tag{71}$$

subject to

$$p_t^c c_t^c + k_{t+1} = R_t^c k_t. \tag{72}$$

Let us guess that

$$v_t(k_t) = E\ln(k_t) + H_t. \tag{73}$$

$$L_t^{c,pt} = \ln(c_t^c) + \beta\left[E\ln(R_t^c k_t - p_t^c c_t^c) + H_{t+1}\right]. \tag{74}$$

The first-order condition is

$$\frac{dL_t^{c,pt}}{dc_t^c} = \frac{1}{c_t^c} - \frac{\beta^c E p_t^c}{R_t^c k_t - p_t^c c_t^c} = 0 \tag{75}$$

$$\frac{1}{c_t^c} = \frac{\beta^c E p_t^c}{R_t^c k_t - p_t^c c_t^c}$$

$$R_t^c k_t - p_t^c c_t^c = \beta^c E p_t^c c_t^c$$

$$(1+\beta^c E) p_t^c c_t^c = R_t^c k_t$$

$$c_t^c = \frac{R_t^c k_t}{(1+\beta^c E) p_t^c}. \tag{76}$$

$$R_t^c k_t - p_t^c c_t^c = R_t^c k_t - \frac{R_t^c k_t}{1+\beta^c E} = \frac{\beta^c E}{1+\beta^c E} R_t^c k_t$$

$$v_t(k_t) = \ln\left(\frac{R_t^c k_t}{(1+\beta^c E) p_t^c}\right) + \beta^c \left[E \ln\left(\frac{\beta^c E}{1+\beta^c E} R_t^c k_t\right) + H_{t+1}\right]$$

$$E\ln(k_t) + H_t = (1+\beta^c E)\ln(k_t) + \ln\left(\frac{R_t^c}{(1+\beta^c E) p_t^c}\right) + \beta^c \left[E \ln\left(\frac{\beta^c E}{1+\beta^c E} R_t^c\right) + H_{t+1}\right]$$

$$E = 1 + \beta^c E \tag{77}$$

$$H_t = \ln\left(\frac{R_t^c}{(1+\beta^c E) p_t^c}\right) + \beta^c \left[E \ln\left(\frac{\beta^c E}{1+\beta^c E} R_t^c\right) + H_{t+1}\right] \tag{78}$$

Thus

$$E = \frac{1}{1-\beta^c}$$

$$1 + \beta^c E = E = \frac{1}{1-\beta^c}$$

$$c_t^c = (1-\beta^c)\frac{R_t^c k_t}{p_t^c} \tag{79}$$

$$k_{t+1} = R_t^c k_t - p_t^c c_t^c = \frac{\frac{\beta^c}{1-\beta^c}}{\frac{1}{1-\beta^c}} R_t^c k_t = \beta^c R_t^c k_t.$$

Thus

$$\begin{aligned} c_{t+1}^c &= (1-\beta^c)\frac{R_{t+1}^c k_{t+1}}{p_{t+1}^c} = (1-\beta^c)\frac{R_{t+1}^c}{p_{t+1}^c}\beta^c R_t^c k_t = \beta^c R_{t+1}^c \left((1-\beta^c)\frac{R_t^c k_t}{p_{t+1}^c}\right) \\ &= \beta^c R_{t+1}^c \frac{p_t^c}{p_{t+1}^c}\left((1-\beta^c)\frac{R_t^c k_t}{p_t^c}\right) \end{aligned}$$

$$c_{t+1}^c = \beta^c R_{t+1}^c \frac{p_t^c}{p_{t+1}^c} c_t^c. \tag{80}$$

For a price-taking capitalist, a constant consumption tax is *not* distortionary. The Euler equation is unchanged.

In the steady state,

$$\beta^c R^c(K) = 1 \tag{81}$$

$$k^l = sw^{at}(K) \tag{82}$$

$$K = \mu k^l + (1-\mu)k$$

$$k = \frac{K - \mu s w^{at}(K)}{1-\mu}. \tag{83}$$

5 The Tradeoff Between Taxing Capital and Labor Income for Capitalists

Paradoxically, while we showed in Section 2 that laborers would not want to tax capital income, the situation for capitalists is actually more complicated.

$$K(\tau_k) = \mu s w_{at}(\tau_k) + (1-\mu)k(\tau_k)$$

Thus

$$k(\tau_k) = \frac{K(\tau_k) - \mu s w_{at}(\tau_k)}{1-\mu}$$

$$\begin{aligned} k'(\tau_k) &= \frac{1}{1-\mu}\left[\frac{1}{\alpha-1}\frac{(\beta^c)^{-1}-1}{(\beta^c)^{-1}-1+\delta(1-\tau_k)}\frac{1}{1-\tau_k}K(\tau_k)\right. \\ &\quad \left. + \mu s \frac{((\beta^c)^{-1}-1)^2}{1-\alpha}\frac{\tau_k}{(1-\tau_k)^2\left[(\beta^c)^{-1}-1+\delta(1-\tau_k)\right]}\frac{K(\tau_k)}{N}\right] \end{aligned}$$

$$\begin{aligned} k'(\tau_k) &= \frac{1}{1-\mu}\frac{1}{\alpha-1}\frac{(\beta^c)^{-1}-1}{(\beta^c)^{-1}-1+\delta(1-\tau_k)}\frac{1}{1-\tau_k}K(\tau_k) \\ &\quad \times \left[1 - \frac{\tau_k}{1-\tau_k}((\beta^c)^{-1}-1)\frac{\mu s}{N}\right] \end{aligned}$$

$$N = \eta\mu\frac{1+\beta^l}{1+\eta\beta^l}$$

$$s = \frac{\eta\beta^l}{1+\eta\beta^l}$$

$$\frac{\mu s}{N} = \frac{\mu\frac{\eta\beta^l}{1+\eta\beta^l}}{\eta\mu\frac{1+\beta^l}{1+\eta\beta^l}} = \frac{\beta^l}{1+\beta^l}$$

$$k'(\tau_k) = \frac{1}{1-\mu}\frac{1}{\alpha-1}\frac{(\beta^c)^{-1}-1}{(\beta^c)^{-1}-1+\delta(1-\tau_k)}\frac{1}{1-\tau_k}K(\tau_k)\left[1-\frac{\tau_k}{1-\tau_k}((\beta^c)^{-1}-1)\frac{\beta^l}{1+\beta^l}\right] \tag{84}$$

Since

$$c^c = ((\beta^c)^{-1}-1)k$$

in the steady state, the capitalist's welfare is strictly increasing in k. Paradoxically, if the capital tax rate gets hiked up high enough, the capital tax will continue to hurt the workers and stop hurting the capitalists. Let τ_k^* satisfy

$$1 = \frac{\tau_k^*}{1-\tau_k^*}((\beta^c)^{-1}-1)\frac{\beta^l}{1+\beta^l},$$

so if $\tau_k > \tau_k^*$, $k'(\tau_k) > 0$.

$$1-\tau_k^* = \tau_k^*((\beta^c)^{-1}-1)\frac{\beta^l}{1+\beta^l}$$

$$\tau_k^* = \frac{1}{1+((\beta^c)^{-1}-1)\frac{\beta^l}{1+\beta^l}}. \tag{85}$$

Note, however, that at such high tax rates the capital stock may be so low that there is no capital left after the saving of the laborers.

$$\begin{aligned} k(\tau_k) &= \frac{K(\tau_k)}{1-\mu}\left(1-\frac{\mu s}{N}\left(J(\tau_k)-\frac{G}{K(\tau_k)}\right)\right) \\ &= \frac{K(\tau_k)}{1-\mu}\left[1-\frac{\beta^l}{1+\beta^l}\left(\frac{1}{\alpha}\frac{(\beta^c)^{-1}-1}{1-\tau_k}+\delta\frac{1-\alpha}{\alpha}+1-(\beta^c)^{-1}-\frac{G}{K(\tau_k)}\right)\right] \end{aligned}$$

Let τ_k^{**} satisfy

$$\frac{\beta^l}{1+\beta^l}\left(\frac{1}{\alpha}\frac{(\beta^c)^{-1}-1}{1-\tau_k^{**}}+\delta\frac{1-\alpha}{\alpha}+1-(\beta^c)^{-1}+G\right) = 1$$

$$\frac{1}{\alpha}\frac{(\beta^c)^{-1}-1}{1-\tau_k^{**}}+\delta\frac{1-\alpha}{\alpha}+1-(\beta^c)^{-1}+G = \frac{1+\beta^l}{\beta^l}$$

$$\frac{1}{\alpha}\frac{(\beta^c)^{-1}-1}{1-\tau_k^{**}} = \frac{1+\beta^l}{\beta^l}$$

6 A Progressive Tax on Consumption

To back out the bargaining power of the respective parties, we need an outside option. We can compute what happens for both types under autarky. For the workers, we can compute what happens with

$$G^l = \frac{2\mu}{1+\mu}G \tag{86}$$

paid for by labor income taxes. For the capitalists, we can compute what happens with

$$G^c = \frac{1-\mu}{1+\mu} G \tag{87}$$

paid for by a lump-sum tax. We will need η for the capitalists' preferences. We can assume that is the same as for the laborers.

In a calibration with $\alpha = 1/3$, $\delta = 1$, $\gamma^c = \gamma^l = 1$, $K/Y = 3$ years and $\kappa = 0.2$, we can only reduce the labor tax rate by 22.8% before wiping out the consumption of capitalists. Laborers would benefit by the equivalent of 6.6% of consumption. In terms of compensating variations, the policy budget line is a straight line. An increase in laborer welfare equivalent to one percentage point of consumption costs capitalists 15 percentage points. Thus the common wisdom that even if you taxed the superrich 100% you would not really make a dent on our fiscal problems is correct. In the baseline model, the capitalists only enjoy 6% of total consumption.

In autarky, we would have

$$K_l = \mu s (1-\tau_n)(1-\alpha)\left(\frac{K_l}{N}\right)^{\alpha}$$

$$\tau_n w_l N = G_l$$

$$\tau_n (1-\alpha)\left(\frac{K_l}{N}\right)^{\alpha} N = G_l$$

$$K_l = \mu s (1-\alpha)\left(\frac{K_l}{N}\right)^{\alpha} - \mu s \frac{G_l}{N}$$

$$K_l = \mu s \left[(1-\alpha)\left(\frac{K_l}{N}\right)^{\alpha} - \frac{G_l}{N}\right]$$

6.1 Capitalist Autarky

For the capitalist, if we have a lump-sum tax in autarky we can simply solve the social planner's problem, which is to solve the Bellman equation

$$v^{sp}(K_t) = \eta \ln(c_t^c) + (1-\eta)\ln(l_t^c) + \beta^c v(K_{t+1}) \tag{88}$$

subject to

$$(1-\mu)c_t^c + K_{t+1} + G^c = F(K_t, (1-\mu)(1-l_t^c)). \tag{89}$$

$$L^{sp} = \eta \ln(c_t^c) + (1-\eta)\ln(l_t^c) + \beta^c v^{sp}(K_{t+1}) + \lambda_t^{sp}\left[K_t^{\alpha}(1-\mu)^{1-\alpha}(1-l_t^c)^{1-\alpha} - (1-\mu)c_t^c - K_{t+1} - G^c\right] \tag{90}$$

$$\frac{\partial L^{sp}}{\partial c_t^c} = \frac{\eta}{c_t^c} - \lambda_t^{sp}(1-\mu) = 0 \tag{91}$$

$$\frac{\partial L^{sp}}{\partial l_t^c} = \frac{1-\eta}{l_t^c} - (1-\alpha)\lambda_t^{sp}(1-\mu)\left(\frac{K_t}{(1-\mu)(1-l_t^c)}\right)^{\alpha} = 0 \tag{92}$$

$$\frac{\partial L^{sp}}{\partial K_{t+1}} = \beta^c \frac{dv^{sp}(K_{t+1})}{dK_{t+1}} - \lambda_t^{sp} = 0 \tag{93}$$

$$\frac{dv^{sp}(K_t)}{dK_t} = \frac{\partial L^{sp}}{\partial K_t} = \alpha \lambda_t^{sp} \left(\frac{K_t}{(1-\mu)(1-l_t^c)} \right)^{\alpha-1} \tag{94}$$

$$\lambda_t^{sp} = \frac{\eta}{(1-\mu)c_t^c}$$

$$\frac{\eta}{(1-\mu)c_t^c} = \beta^c \frac{dv^{sp}(K_{t+1})}{dK_{t+1}}$$

$$\frac{\eta}{(1-\mu)c_t^c} = \alpha \beta^c \lambda_{t+1}^{sp} \left(\frac{K_{t+1}}{(1-\mu)(1-l_{t+1}^c)} \right)^{\alpha-1}$$

$$\frac{\eta}{c_t^c} = \alpha \beta^c \frac{\eta}{c_{t+1}^c} \left(\frac{K_{t+1}}{(1-\mu)(1-l_{t+1}^c)} \right)^{\alpha-1}$$

$$\frac{c_{t+1}^c}{c_t^c} = \beta^c R_{t+1} \tag{95}$$

In the steady state, we must then have

$$R_c = (\beta^c)^{-1}$$

$$R_c = \alpha \left(\frac{K_c}{N_c} \right)^{\alpha-1}$$

$$\frac{K_c}{N_c} = \left(\frac{R_c}{\alpha} \right)^{\frac{1}{\alpha-1}} \tag{96}$$

$$w_c = (1-\alpha) \left(\frac{K_c}{N_c} \right)^{\alpha} = (1-\alpha) \left(\frac{R_c}{\alpha} \right)^{\frac{\alpha}{\alpha-1}} \tag{97}$$

$$\frac{K_c}{Y_c} = \frac{K_c}{K_c^{\alpha} N_c^{1-\alpha}} = \left(\frac{K_c}{N_c} \right)^{1-\alpha} = \frac{\alpha}{R_c}$$

$$\frac{1-\eta}{l^c} = \lambda^{sp}(1-\mu) w_c$$

$$\frac{1-\eta}{l^c} = \frac{\eta}{(1-\mu)c^c}(1-\mu) w_c$$

$$c^c = \frac{\eta}{1-\eta} w_c l^c$$

$$C^c = (1-\mu) c^c$$

$$C^c = \frac{\eta}{1-\eta} w_c (1-\mu) l^c$$

$$C^c + K^c + G^c = F(K^c, (1-\mu)(1-l^c))$$

$$\frac{\eta}{1-\eta} w_c (1-\mu) l^c + \frac{K^c}{N^c}(1-\mu)(1-l^c) + G^c = \frac{Y^c}{N^c}(1-\mu)(1-l^c)$$

Let

$$g = \frac{G^c}{1-\mu}$$

$$\frac{\eta}{1-\eta} w_c l^c + \frac{K^c}{N^c}(1-l^c) + g = \frac{Y^c}{N^c}(1-l^c)$$

$$\left[\frac{\eta}{1-\eta} w_c - \frac{K^c}{N^c} + \frac{Y^c}{N^c}\right] l^c = \frac{Y^c}{N^c} - \frac{K^c}{N^c} - g$$

$$l^c = \frac{\frac{Y^c}{N^c} - \frac{K^c}{N^c} - g}{\frac{Y^c}{N^c} - \frac{K^c}{N^c} + \frac{\eta}{1-\eta} w_c}$$

Using Nash bargaining, we find that the economy with no consumption tax on capitalists is optimal if the capitalists have roughly 70% of the bargaining power. Going to autarky would cause laborers to lose the equivalent of 12% of consumption. It would cause capitalists to lose the equivalent of 99.5% of consumption. Of course, this raises the paradox of why, with so much bargaining power, capitalists have failed to eliminate taxes on capital, which would benefit everyone.

7 Baseline Calibration

We calibrate $K/Y = 3$ and $\kappa = 0.2$ with $\alpha = 1/3$ and $\delta = 1$. To calibrate the current tax structure, let us assume there are only type-independent taxes on capital and labor. To calibrate these we assume $G/Y = 0.15$ and we set $\tau^k = \tau^{k,c} = \tau^{k,l} = 0.27$, which is the average federal income tax rate of the 0.1% reported by the IRS in 2016. Then we get $\beta^c = 0.3699$ (0.9674 in annual terms) and $\beta^l = 0.1601$ (0.9408 in annual terms). Then $\tau_k^* = 0.81$, but this is high enough that the capitalist would not have any capital.

As in Feigenbaum (2018), eliminating the capital tax improves welfare for both the laborers and capitalists. The former enjoy a welfare gain equivalent to 1.8% of baseline consumption. The capitalists enjoy a much larger gain since consumption increases by 177%.

8 Conclusion

The next step obviously is to repeat the exercises here for the case of a segregated economy model with price-setting capitalists. We get several analytic results in this price-taking case, but those proofs are facilitated by some strikingly implausible predictions, especially the prediction that the capital-output ratio is completely insensitive to all policy variables except the tax rate on capital income. When capitalists are price-takers, wealth inequality has no effect on the welfare of laborers. In contrast, when capitalists are price-setters, Feigenbaum (2018) shows that Smith (1776) distortions arise. Capitalists can inefficiently increase the return on capital and lower wages by colluding to reduce their saving. The finding that both capitalists and laborers are hurt more by a tax on capital income than by a tax on labor income may depend on the absence of a channel through which wealth inequality can hurt workers.

9 General CRRA Utility for Laborers

A laborer born at time t will maximize

$$U_t^l = u^l(c_t^l, l_t) + \beta^l u^l(c_{t+1}^l, 1) \tag{98}$$

subject to

$$p_t^l c_{t,0}^l + k_{t+1}^l = w_t^{at}(1 - l_t) \tag{99}$$

$$p_{t+1}^l c_{t+1,1}^l = R_t^l k_{t+1}^i. \tag{100}$$

The lifetime budget constraint is

$$p_t^l c_{t,0}^l + \frac{p_{t+1}^l c_{t+1,1}^l}{R_t^l} = w_t^{at}(1 - l_t). \tag{101}$$

The laborer's Lagrangian is

$$L_t^l = u^l(c_t^l, l_t) + \beta^l u^l(c_{t+1}^l, 1) + \lambda_t \left[w_t^{at}(1 - l_t) - p_t^l c_{t,0}^l - \frac{p_{t+1}^l c_{t+1,1}^l}{R_t^l} \right]. \tag{102}$$

$$\frac{\partial L_t^l}{\partial c_{t,0}^l} = \eta (c_{t,0}^l)^{\eta(1-\gamma^l)-1} (l_t)^{(1-\eta)(1-\gamma^l)} - \lambda_t p_t^l = 0 \tag{103}$$

$$\frac{\partial L_t^l}{\partial c_{t+1,1}^l} = \beta^l \eta (c_{t+1,1}^l)^{\eta(1-\gamma^l)-1} - \frac{\lambda_t p_{t+1}^l}{R_t^l} = 0 \tag{104}$$

$$\frac{\partial L_t^l}{\partial l_t} = (1-\eta)(c_{t,0}^l)^{\eta(1-\gamma^i)} l_t^{(1-\eta)(1-\gamma^l)-1} - \lambda_t w_t^{at} = 0 \tag{105}$$

$$\eta (c_{t,0}^l)^{\eta(1-\gamma^l)-1} (l_t)^{(1-\eta)(1-\gamma^l)} = \lambda_t p_t^l$$

$$(1-\eta)(c_{t,0}^l)^{\eta(1-\gamma^i)} l_t^{(1-\eta)(1-\gamma^l)-1} = \lambda_t w_t^{at}$$

A General CRRA Utility for Capitalists

A capitalist solves the Bellman equation

$$v_t(k_t) = u^c(c_t^c) + \beta^c v_{t+1}(k_{t+1}) \tag{106}$$

subject to

$$p_t^c c_t^c + k_{t+1} = R_t k_t. \tag{107}$$

Let us guess that

$$v_t(k_t) = E_t u^c(k_t) \tag{108}$$

$$L_t^c = u^c(c_t^c) + \beta^c E_{t+1} u^c(R_t k_t - p_t^c c_t^c). \tag{109}$$

The first-order condition is

$$\frac{dL_t^c}{dc_t^c} = (c_t^c)^{-\gamma^c} - \beta^c p_t^c E_{t+1}(R_t k_t - p_t^c c_t^c)^{-\gamma^c} = 0 \tag{110}$$

$$(c_t^c)^{-\gamma^c} = \beta^c p_t^c E_{t+1}(R_t k_t - p_t^c c_t^c)^{-\gamma^c}$$

$$c_t^c = (\beta^c p_t^c E_{t+1})^{-1/\gamma^c} (R_t k_t - p_t^c c_t^c)$$

$$p_t^c c_t^c = \left(\beta^c (p_t^c)^{1-\gamma^c} E_{t+1}\right)^{-1/\gamma^c} (R_t k_t - p_t^c c_t^c)$$

$$\left(1 + \left(\beta^c (p_t^c)^{1-\gamma^c} E_{t+1}\right)^{-1/\gamma^c}\right) c_t^c = \left(\beta^c (p_t^c)^{1-\gamma^c} E_{t+1}\right)^{-1/\gamma^c} R_t k_t$$

$$c_t^c = \frac{R_t^c k_t}{(1 + \left(\beta^c (p_t^c)^{1-\gamma^c} E_{t+1}\right)^{1/\gamma^c}) p_t^c}. \tag{111}$$

$$R_t^c k_t - p_t^c c_t^c = R_t^c k_t - \frac{R_t^c k_t}{1+\beta E} = \frac{\beta E}{1+\beta E} R_t^c k_t$$

$$v_t(k_t) = \ln\left(\frac{R_t^c k_t}{(1+\beta E)p_t^c}\right) + \beta\left[E \ln\left(\frac{\beta E}{1+\beta E} R_t^c k_t\right) + H_{t+1}\right]$$

$$E\ln(k_t) + H_t = (1+\beta E)\ln(k_t) + \ln\left(\frac{R_t^c}{(1+\beta E)p_t^c}\right) + \beta\left[E\ln\left(\frac{\beta E}{1+\beta E} R_t^c\right) + H_{t+1}\right]$$

$$E = 1 + \beta E \tag{112}$$

$$H_t = \ln\left(\frac{R_t^c}{(1+\beta E)p_t^c}\right) + \beta\left[E\ln\left(\frac{\beta E}{1+\beta E} R_t^c\right) + H_{t+1}\right] \tag{113}$$

Thus

$$E = \frac{1}{1-\beta}$$

$$1 + \beta E = E = \frac{1}{1-\beta}$$

$$c_t^c = (1-\beta)\frac{R_t^c k_t}{p_t^c} \tag{114}$$

$$k_{t+1} = R_t^c k_t - p_t^c c_t^c = \frac{\frac{\beta}{1-\beta}}{\frac{1}{1-\beta}} R_t^c k_t = \beta R_t^c k_t.$$

Thus

$$\begin{aligned} c_{t+1}^c &= (1-\beta)\frac{R_{t+1}^c k_{t+1}}{p_{t+1}^c} = (1-\beta)\frac{R_{t+1}^c}{p_{t+1}^c}\beta R_t^c k_t = \beta R_{t+1}^c \left((1-\beta)\frac{R_t^c k_t}{p_{t+1}^c}\right) \\ &= \beta R_{t+1}^c \frac{p_t^c}{p_{t+1}^c}\left((1-\beta)\frac{R_t^c k_t}{p_t^c}\right) \end{aligned}$$

$$c_{t+1}^c = \beta R_{t+1}^c \frac{p_t^c}{p_{t+1}^c} c_t^c. \tag{115}$$

For a price-taking capitalist, a constant consumption tax is *not* distortionary. The Euler equation is unchanged.In the steady state,

$$\beta R^c(K_{pt}) = 1$$

$$k_{pt}^l = sw^{at}(K_{pt})$$

$$K^{pt} = \mu k_{pt}^l + (1-\mu)k_{pt}$$

$$k_{pt} = \frac{K^{pt} - \mu s w^{at}(K_{pt})}{1-\mu}.$$

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