Taxes, Imperfect Capital Markets, and Illegal Immigration*

James Feigenbaum†

Utah State University

Work in Progress
Very Preliminary

April 4, 2017

Abstract

Despite widespread concern about illegal immigrants among laypeople and politicians, in the very simplest adverse immigration models where legal resident households must choose between two productivity levels in an otherwise frictionless overlapping-generations model, deporting a large fraction of low-productivity workers has no long-term impact on welfare. Here we show that illegal immigration can have long-term welfare effects if taxes and benefits are introduced. Because illegal immigrants do not receive Social Security benefits, they work more and save more than low-educated legal residents. This increase in the low-educated labor supply incentivizes more legal residents to invest in high education, and higher-educated households also save more. Together, the increased saving of illegal immigrants and high-educated labor results in a higher equilibrium capital stock. However, if public spending leads to a high enough increase in taxes on low-educated legal residents, the effect of the higher capital stock can be countervailed. If the elasticity of public goods (other than Social Security) with respect to population is less than 0.9, legal residents enjoy higher welfare in an equilibrium where 5% of the population is an illegal immigrant and pays half the taxes of the corresponding legal resident than in the corresponding equilibrium where those 5% are removed from the population. If the elasticity of public goods with respect to the legal population is more than 0.1, legal residents will also enjoy higher welfare in the equilibrium with 5% illegal immigrants than in the corresponding equilibrium where those 5% are legalized.

JEL Classification: E21, E24, F22

Keywords: adverse immigration model, overlapping-generations model, occupational choice, distortionary taxes, tax evasion, remittances

Immigration has become a major issue worldwide with the refugee crisis in Europe among other events. Illegal immigration, in particular, was a central issue in the 2016 U.S. presidential election. Feigenbaum (2015) endeavored to present the simplest possible model in which immigration might conceivably have adverse effects and showed that a mass deportation of illegal immigrants would have a detrimental effect on high-productivity workers while conferring, at most, only a negligible improvement to low-productivity

*I would like to thank Sandra Orozco-Aleman and Annie Fang Yang for their help with the calibration. I also want to thank Yamin Ahmad, Rick Evans, Scott Findley, Aspen Gorry, Michael Isenberg, Hamid Mohtadi, Kerk Phillips, Cathy Yuhan Xue, and participants at the BYU/USU Macro Workshop, the University of Wisconsin-Milwaukee, and the University of Wisconsin-Whitewater for their comments and suggestions.

†Corresponding author: J.Feigen@aggiemail.usu.edu.

workers. Here I extend that model to incorporate a public sector in order to study the welfare effects for legal residents of various proposals regarding the tax treatment of illegal immigrants.

The purpose of the earlier paper was to test the widespread presumption among laypeople and politicians that the mere presence of illegal immigrants in the labor market can account for the sluggishness of wage growth during the last several decades in America. In this elementary model, legal residents and illegal immigrants had the same preferences and productivity ceteris paribus. There was only one practical difference between the two groups: illegal immigrants could not borrow to finance higher education, practical difference between legal residents and illegal immigrants was that the latter could not borrow to pay for high education while legal residents could if they chose to do so. Low-educated legal residents and illegal immigrants behave exactly the same in Feigenbaum (2015) and together constitute the low-educated labor supply. The elasticity of low-educated wages with respect to low-educated labor in this frictionless model is the share of output going to other factors of production divided by the elasticity of substitution between low-educated labor and those other factors.[1] In order for this elasticity to be greater than one, low-educated labor would have to be complementary to high-educated labor and capital, which is clearly counterfactual.[2] Thus, even if the entire population of illegal immigrants, making up roughly 5% of the labor supply, was removed from the United States, this could yield at most a 5% increase in low-educated wages, and that is an extremely conservative upper bound. The baseline calibration of Feigenbaum (2015) generates an increase of less than 1% from this experiment.

I conclude from this that, if illegal immigrants are significantly reducing low-educated wages, it must be because of some friction resulting from their illegality and not simply the consequence of their numbers.[3] The most basic friction that can be incorporated into the model of Feigenbaum (2015) is differences in the tax treatment of illegal immigrants and legal residents.[4] Once taxes are introduced, we can also incorporate Social Security. I generally find that a deportation of illegal immigrants would have a long-run negative welfare effect on legal residents.

The mechanism by which illegal immigrants affect legal-resident welfare is essentially the same as the mechanism that generates the transition dynamics in Feigenbaum (2015). The presence of illegal immigrants in the labor force incentivizes legal residents to invest more in higher education, resulting in higher output and a bigger capital stock. It would be natural to intuit that the lower taxes paid by illegal immigrants are responsible for making this effect permanent in the present model. However, increasing the efficiency of tax collection for illegal immigrants can actually increase the proportion of legal residents who become high educated, so that cannot be the answer. Of course, taxes have an ambiguous effect on labor supply because a change in taxes has both income and substitution effect.

Granting a household Social Security benefits, on the other hand, is a pure income effect, and the long-term effects of illegal immigrants can primarily be attributed to them not receiving benefits, which leads them to work more hours than low-educated legal residents. This increase in the low-educated labor supply increases the education premium, incentivizing more legal residents to become high educated, resulting in a higher capital stock and higher low-educated wages.

Political debate about immigration reform has also thrown some light on the burgeoning issue of remittances–transfers of cash between households in different countries, usually from expatriate workers in wealthy coun-

[1] Note this is the reciprocal of the usual elasticity of demand for low-educated labor.

[2] Some low-skilled jobs like janitorial services may be necessary for high-skilled labor and capital to be productive and so would be complementary. However, most low-skilled workers today fear that they will be replaced either by foreigners or by technology, which is only possible if their labor is a substitute for capital.

[3] In a converse paper, Krause (2017) considers the threat of emigration by high-skilled households when there is a redistributive tax (and everyone is legal). He finds an analogous result that in the short run there are some benefits from the exodus and in the long run there is no effect.

[4] Illegal immigrants presumably face more severe search frictions than legal residents, but adding search frictions to the current three-period model would be too cumbersome. See Casarico et al (2016) for work that pursues that approach instead of the general-equilibrium approach of this paper.

tries to family members back in the home country. Until recently, remittances were a negligible component of the international flow of funds, but today they are of the same order of magnitude as net factor payments from abroad. Since net remittances into a country are included in the standard formula for a country's current account balance, some have suggested that remittances to other countries are draining away the United States' wealth. In fact, this is a gross misunderstanding. Remittances are included in the computation of the current account to cancel out the effect that remittances have on net exports since accounting identities require one dollar of remittance sent to another country to exactly crowd in one dollar of net exports. Thus remittances have no effect on the current account.

Here we will show that, with commonplace preferences, taxes on remittances have a neutral effect on native households. To be precise, if only immigrant households value remittances, for typical preferences where the elasticity of substitution between remittances, consumption, and leisure is constant, a tax on remittances will not alter the immigrant household's labor supply or lifecycle pattern of saving. Thus a tax on remittances cannot affect resident households by changing either the supply of labor or the demand for capital. Labor and capital markets will be unaffected, so, ceteris paribus, a tax on remittances will not change any variables that affect the welfare of native households that do not care about remittances.

But if remittances are truly neutral, that would suggest they are a ripe target for taxation. Perhaps we can substitute taxes on remittances for taxes on labor, in which case a reduced tax on labor should improve the welfare of native households. In this generalized version of Feigenbaum (2015), we can compute the optimal revenue-neutral mix of taxes on labor and remittances, using the lifetime utility of resident households, which must be the same in equilibrium for all legal residents, as our social welfare function. Of course, legal residents of the United States include both "native" households that have been in the United States for several generations and recent immigrants, so a representative-agent of legal resident households ought to care about remittances too, though perhaps not as much illegal immigrant households. Allowing for this, remittances will be only approximately neutral.even if we hold taxes on labor fixed. Even a legal resident household will not see a costless tradeoff of labor taxes for taxes on remittances.[5]

1 The Model

Suppose we generalize the model of Feigenbaum (2015) by including both remittances and leisure in the utility function. Thus there are three types of households. 0 is a (low-educated) illegal immigrant, 1 is a low-educated legal resident, and 2 is a high-educated legal resident.

We consider only steady-state equilibria. Let $n(s)$ be the measure of agents of type $s \in \{0, 1, 2\}$ such that

$$\sum_{s=0}^{2} n(s) = P, \tag{1}$$

where P is the population (usually normalized to 1). While $n(0) = \zeta$ will be exogenous, $n(1)$ and $n(2)$ will be determined so the utility of low-educated and high-educated native households is the same.

A typical variable will be of the form $x_t(s)$, where t represents the age of a household and s represents the type. A household of type s and age t will be endowed with $e_t(s) \geq 0$ efficiency units of labor. Since the efficiency profile is likely to peak in middle age, we normalize it by $e_1(s) = 1$ for each type. Also, households seeking education have $e_0(2) = 0$ when they are young. The wage per efficiency unit of a household of type

[5] One candidate in the 2016 presidential election suggested we could confiscate remittances without any cost to native households. This is absurd since, in a modern world with many different channels for sending money abroad, a large tax on remittances would be hard to collect and easy to avoid. It also demonstrates, though, why it is essential to introduce some cost to taxing remittances for legal resident households.

s will be $w(s)$, and the gross interest rate on saving is R. We assume that $w(0) = w(1)$ and $e_t(0) = e_t(1)$ for $t = 0, ..., 2$, so there is no technological difference between an illegal immigrant and a low-educated worker.

The consumption of a household at age t will be denoted by $c_t(s)$, leisure by $l_t(s)$, remittances by $m_t(s)$, and the saving of such a household by $b_{t+1}(s)$. The discount factor is $\beta > 0$. The period utility function

$$u_s(c, l, m)$$

will depend on consumption c, leisure l, and possibly remittances m. We assume there is no difference in preferences between a low- and high-educated native household, so $u_1(c, l, m) = u_2(c, l, m)$ for all permissible (c, l, m).

We now generalize the model to limit the access of illegal immigrants to capital markets. Let us define the effective interest rate $R_{eff}(i)$ where $R_{eff}(1) = R_{eff}(2) = R$ and

$$R_{eff}(0) = \omega R + (1 - \omega)(1 - \delta),$$

where $\omega \in [0, 1]$ measures the access of illegal immigrants to capital markets. For $\omega = 0$, illegal immigrants can only store consumption as durable goods that depreciate at the rate δ. For $\omega > 0$, we can imagine that a fraction ω are able to invest in capital while $1 - \omega$ cannot, and the illegal immigrant household is a representative agent of illegal immigrants.

The problem of a low-educated native is then to maximize

$$U(1) = \sum_{t=0}^{2} \beta^t u_1(c_t(1), l_t(1), m_t(1))$$

subject to

$$c_0(1) + (1 + \tau_m)m_0(1) + b_1(1) = (1 - \tau_y(1))w(1)(1 - l_0(1))e_0(1) \quad (2)$$

$$c_1(1) + (1 + \tau_m)m_1(1) + b_2(1) = (1 - \tau_y(1))w(1)(1 - l_1(1))e_1(1) + Rb_1(1) \quad (3)$$

$$c_2(1) + (1 + \tau_m)m_2(1) = (1 - \tau_y(1))w(1)(1 - l_2(1))e_2(1) + Rb_2(1) + B(1), \quad (4)$$

$$0 \leq l_t(1) \leq 1 \quad t = 0, 1, 2$$

$$\begin{aligned} b_t(1) &\geq 0 \quad t = 1, 2, 3 \\ c_t(1), m_t(1) &\geq 0 \quad t = 0, 1, 2. \end{aligned}$$

where η is the weight of consumption and $1 - \eta$ is the weight on leisure in the CRRA aggregator. Here $\tau_y(s)$ is the tax on labor income, which may depend on the type; τ_m is the tax on remittances; and $B(s)$ is the Social Security benefit received by a household of type $s \in \{1, 2\}$.

The problem of a high-educated native is to maximize

$$U(2) = \sum_{t=0}^{2} \beta^t u_2(c_t(2), l_t(2), m_t(2))$$

subject to

$$c_0(2) + (1 + \tau_m)m_0(2) + b_1(2) + \tau_e = 0 \quad (5)$$

$$c_1(2) + (1 + \tau_m)m_1(2) + b_2(2) = (1 - \tau_y(2))w(2)(1 - l_1(2))e_1(2) + Rb_1(2) \quad (6)$$

$$c_2(2) + (1 + \tau_m)m_2(2) = (1 - \tau_y(2))w(2)(1 - l_2(2))e_2(2) + Rb_2(2) + B(2). \quad (7)$$

$$l_0(2) = l_e$$

$$0 \leq l_t(2) \leq 1 \quad t = 1, 2$$
$$b_1(2) \geq -(\tau_e + c_{edu}) \tag{8}$$

$$\begin{aligned} b_2(2), b_2(3) &\geq 0 \\ c_t(2), m_t(2) &\geq 0 \quad t = 0, 1, 2. \end{aligned}$$

Here, τ_e is tuition for higher education and c_{edu} is the maximum amount of consumption that a household seeking higher education can finance through borrowing. The leisure of a young household seeking education, l_e, is exogenous.

The utility of an illegal immigrant is

$$U(0) = \sum_{t=0}^{2} \beta^t u_0 \left(c_t(0), l_t(0), m_t(0) \right).$$

The budget constraints for an immigrant are

$$c_0(0) + (1 + \tau_m) m_0(0) + b_1(0) = w(1)(1 - \tau_y(0))(1 - l_0(0)) e_0(1) \tag{9}$$

$$c_1(0) + (1 + \tau_m) m_1(0) + b_2(0) = (1 - \tau_y(0)) w(1)(1 - l_1(1)) e_1(1) + R_{eff}(0) b_1(0) \tag{10}$$

$$c_2(0) + (1 + \tau_m) m_2(0) = (1 - \tau_y(0)) w(1)(1 - l_2(1)) e_2(1) + R_{eff}(0) b_2(0). \tag{11}$$

$$b_t(0) \geq 0 \quad t = 1, 2, 3$$
$$0 \leq l_t(0) \leq 1 \quad t = 1, 2$$
$$c_t(0), m_t(0) \geq 0 \quad t = 0, 1, 2.$$

We assume illegal immigrants have the same efficiency profile and receive the same wage as low-educated households. Note that illegal immigrants do not receive Social Security benefits.

Low-educated labor is

$$N(1) = \sum_{s=0}^{1} n(s) \sum_{t=0}^{2} e_t(1)(1 - l_t(s)) \tag{12}$$

while skilled labor at t is

$$N(2) = n(s) \sum_{t=1}^{2} e_t(s)(1 - l_t(2)). \tag{13}$$

The capital equilibrium condition is[6]

$$K = \omega n(0) \sum_{t=1}^{2} b_t(0) + \sum_{s=1}^{2} n(s) \sum_{t=1}^{2} b_t(s). \tag{14}$$

Aggregate consumption is

$$C = \sum_{s=0}^{2} n(s) \sum_{t=0}^{2} c_t(s). \tag{15}$$

[6] We are assuming here that there are no capital flows, which is somewhat at odds with our assumption that there are remittance flows out of the country. We are assuming for simplicity that the economy is closed by preference rather than by physical or legal barriers. The countries that immigrants are coming from and that remittances are going to do not produce goods that this country is interested in buying. Since there is no trade, the only flow of goods to the other country is in the form of unilateral remittances.

Aggregate remittances is

$$M = \sum_{s=0}^{2} n(s) \sum_{t=0}^{2} m_t(s). \tag{16}$$

The production function is

$$Y = F(K, N(1), N(2)), \tag{17}$$

where F exhibits constant returns to scale. Firms behave competitively so factor prices are

$$R = F_K(K, N(1), N(2)) + 1 - \delta \tag{18}$$

$$w(1) = F_{N(1)}(K, N(1), N(2)) \tag{19}$$

$$w(2) = F_{N(2)}(K, N(1), N(2)). \tag{20}$$

The government budget constraint is

$$\sum_{s=0}^{2} n(s)w(s)\tau_y(s) \sum_{t=0}^{2} e_t(s)(1 - l_t(s)) + \tau_m M = G + \sum_{s=1}^{2} n(s)B(s), \tag{21}$$

where G is government spending on public goods and services. We will assume for simplicity that benefits take the form

$$B(s) = \rho(s)w(s)$$

where $\rho(s) > 0$ is the replacement rate.[7]

The income-expenditure identity for this model is

$$Y = C + I + M + G, \tag{22}$$

so net exports equal remittances in this model. The current account is defined as net exports plus net factor payments from abroad, which are zero here, minus remittances, which vanishes in this model since there is no capital flow in the model. Note, however, that even if we included capital flows in the model remittances would contribute one for one to net exports so remittances would vanish from the current account. Thus the common concern that remittances are draining wealth from rich countries such as the United States is a misconception.

2 Household Behavior

To solve the problem of each household, it is helpful to decompose the household's problem into an intertemporal problem and a series of intratemporal problems. Let us define the total expenditures on goods of a household of age t and type s as

$$E_t(s) = \begin{cases} c_t(s) + (1+\tau_m)m_t(s) + (1-\tau_s)w(s)e_t(s)l_t(s) & s = 1,2 \\ c_t(0) + (1-\tau_s)w(1)e_t(1)l_t(0) + (1+\tau_m)m_t(0) & s = 0 \end{cases}. \tag{23}$$

Let us also define the income endowment $y_t(s)$ of a household of age t. For an illegal immigrant,

$$y_t(0) = (1 - \tau_0)w(1)e_t(1). \tag{24}$$

[7] $\rho(s)$ is not the replacement rate as we usually define it since that would be defined relative to income rather than to the wage.

For a low-educated native,

$$y_t(1) = \begin{cases} (1-\tau_1)w(1)e_t(1) & t = 0,1 \\ (1-\tau_1)w(1)e_2(1) + B(1) & t = 2 \end{cases} \tag{25}$$

For a high-educated native,

$$y_t(2) = \begin{cases} -\tau_e & t = 0 \\ (1-\tau_2)w(2)e_1(2) & t = 1 \\ (1-\tau_2)w(2)e_2(2) + B(2) & t = 2 \end{cases}.$$

Note here that households treat the Social Security benefit $B(i)$ as exogenous, although, at least in principle, the benefit depends on the household's choice of how much to work.[8]

Then the budget constraints for an out-of-school household of type s can be written

$$E_t(s) + b_{t+1}(s) = y_t(s) + R_{eff}(s)b_t(s) \quad t = 0,1,2 \tag{26}$$

$$b_0(s) = 0 \tag{27}$$

$$b_{t+1}(s) \geq 0, \quad t = 0,1,2. \tag{28}$$

For $E \geq 0$, let us define the intratemporal problem for an out-of-school household as

$$V_s(E, p_m, p_l) = \max u_s(c, l, m) \tag{29}$$

subject to

$$\begin{aligned} c + p_m m + p_l l &= E \\ 0 &\leq l \leq 1. \\ m &\geq 0 \end{aligned}$$

For $E \in [0, \tau_e + c_{edu}]$, let the intratemporal problem for an in-school household be

$$V_e(E, p_m) = \max u_2(c, l_e, m)$$

subject to

$$c + p_m m = E.$$

Then the intertemporal problem will be

$$\max \sum_{t=0}^{2} \beta^t V_s(E_t, 1+\tau_m, (1-\tau_y(s))w(s)e_t(s)) \tag{30}$$

subject to (26)-(28) for a household of types 0 or 1. For a household of type 2, the intertemporal problem will be

$$V_e(E_0, p_m) + \sum_{t=1}^{2} \beta^t V_2(E_t, 1+\tau_m, (1-\tau_y(2))w(s)e_t(s)) \tag{31}$$

[8] This is necessary to decompose the problem into an intertemporal and intratemporal problem. While I know many people who think hard about how the date of their retirement will affect their Social Security benefit, I am not aware of anyone, including myself, who has considered the linkage between their work hours and their future Social Security benefits, so this seems a reasonable assumption to me.

subject to (26)-(27) and

$$\begin{array}{rcl} b_1(2) & \geq & -c_{edu} - \tau_e \\ b_{t+1}(2) & \geq & 0 \quad t = 1, 2. \end{array} \tag{32}$$

Let us suppose the utility function has the form

$$u_s(c, l, m) = g\left((c^{1-\chi_s} (m + \mu_s)^{\chi_s})^{\eta_s} l^{1-\eta_s} \right) \tag{33}$$

for some strictly increasing, strictly concave function g, where $\eta_s, \chi_s \in [0, 1]$ and $\mu_s \geq 0$. We allow for the offset μ_s so that there is no Inada condition for remittances. A household that is poor enough will not contribute to remittances. Then the solution to the intratemporal problem is as follows.[9] If $p_l \geq \frac{1-\eta_s}{\eta_s} \frac{p_m \mu_s}{\chi_s}$, the (out-of-school) value function is

$$V_s(E, p_l, p_m) = \begin{cases} g\left(\kappa_s^{II} \frac{E^{1-\chi_s \eta_s}}{p_l^{1-\eta_s}} \right) & E \leq \frac{1-\eta_s \chi_s}{\eta_s \chi_s} p_m \mu_s \\ g\left(\frac{\kappa_s^{IV}}{p_m^{\eta_s \chi_s} p_l^{1-\eta_s}} (E + p_m \mu_s) \right) & \frac{1-\eta_s \chi_s}{\eta_s \chi_s} p_m \mu_s < E < \frac{p_l}{1-\eta_s} - p_m \mu_s \\ g\left(\kappa_s^{III} \frac{(E - p_l + p_m \mu_s)^{\eta_s}}{p_m^{\chi_s \eta_s}} \right) & \frac{p_l}{1-\eta_s} - p_m \mu_s \leq E \end{cases} \tag{34}$$

If $p_l < \frac{1-\eta_s}{\eta_s} \frac{p_m \mu_s}{\chi_s}$, the value function is

$$V_s(E, p_l, p_m) = \begin{cases} g\left(\kappa_s^{II} \frac{E^{1-\chi_s \eta_s}}{p_l^{1-\eta_s}} \right) & E < \left[1 + (1 - \chi_s) \frac{\eta_s}{1-\eta_s} \right] p_l \\ g(\varkappa_s^I (E - p_l)^{\eta_s (1-\chi_s)}) & \left[1 + (1 - \chi_s) \frac{\eta_s}{1-\eta_s} \right] p_l \leq E \leq \frac{(1-\chi_s)}{\chi_s} p_m \mu_s + p_l \\ g\left(\kappa_s^{III} \frac{(E - p_l + p_m \mu_s)^{\eta_s}}{p_m^{\chi_s \eta_s}} \right) & \frac{(1-\chi_s)}{\chi_s} p_m \mu_s + p_l < E \end{cases} \tag{35}$$

The multiplicative factors κ_s^I, κ_s^{II}, κ_s^{III}, and κ_s^{IV} are defined as

$$\kappa_s^I = \mu_s^{\chi_s \eta_s} \tag{36}$$

$$\kappa_s^{II} = \mu_s^{\chi_s \eta_s} \left((1 - \chi_s) \eta_s \right)^{\eta_s (1-\chi_s)} (1 - \eta_s)^{1-\eta_s} \left(\frac{1}{1 - \chi_s \eta_s} \right)^{1-\chi_s \eta_s} \tag{37}$$

$$\kappa_s^{III} = \left(\frac{\chi_s}{p_m} \right)^{\chi_s \eta_s} (1 - \chi_s)^{(1-\chi_s)\eta_s} \tag{38}$$

$$\kappa_s^{IV} = (\eta_s (1 - \chi_s))^{\eta_s (1-\chi_s)} \left(\frac{\eta_s \chi_s}{p_m} \right)^{\eta_s \chi_s} \left(\frac{1 - \eta_s}{p_l} \right)^{1-\eta_s}. \tag{39}$$

In Appendix B, we show that V_s is strictly concave and continuously differentiable in E. If g is constant relative risk aversion (CRRA), so for $\gamma > 0$,

$$g(x) = \begin{cases} \ln x & \gamma = 1 \\ \frac{1}{1-\gamma} x^{1-\gamma} & \gamma \neq 1 \end{cases}, \tag{40}$$

then there is also an analytic expression for the inverse of $\partial V_s / \partial E$, which we can use to solve the intertemporal problem.

[9]The derivation can be found in Appendix B.

For a high-educated household still in school,

$$V_e(E,p_m) = \begin{cases} g\left(E^{\eta_2(1-\chi_2)}\mu_2^{\eta_2\chi_2} l_e^{1-\eta_2}\right) & E \le \frac{1-\chi_2}{\chi_2}p_m\mu_2 \\ g\left((1-\chi_2)^{\eta_2(1-\chi_2)}\left(\frac{\chi_2}{p_m}\right)^{\eta_2\chi_2} l_e^{1-\eta_2}(E+p_m\mu_2)^{\eta_2}\right) & E > \frac{1-\chi_2}{\chi_2}p_m\mu_2 \end{cases}. \tag{41}$$

This too is strictly concave and continuously differentiable in E, and $\partial V_e/\partial E$ has an analytically derivable inverse if g is CRRA.

The corresponding behavioral functions areIf $p_l \ge \frac{1-\eta_s}{\eta_s}\frac{p_m\mu_s}{\chi_s}$, the consumption function is

$$c(E,p_l,p_m;s) = \begin{cases} \frac{(1-\chi_s)\eta_s}{1-\chi_s\eta_s}E & E \le \frac{1-\eta_s\chi_s}{\eta_s\chi_s}p_m\mu_s \\ \eta_s(1-\chi_s)[E+p_m\mu_s] & \frac{1-\eta_s\chi_s}{\eta_s\chi_s}p_m\mu_s < E < \frac{p_l}{1-\eta_s} - p_m\mu_s \\ (1-\chi_s)[E-p_l+p_m\mu_s] & \frac{p_l}{1-\eta_s} - p_m\mu_s \le E \end{cases} \tag{42}$$

If $p_l < \frac{1-\eta_s}{\eta_s}\frac{p_m\mu_s}{\chi_s}$, the consumption function is

$$c(E,p_l,p_m;s) = \begin{cases} \frac{(1-\chi_s)\eta_s}{1-\chi_s\eta_s}E & E < \left[1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right]p_l \\ E-p_l & \left[1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right]p_l \le E \le \frac{(1-\chi_s)}{\chi_s}p_m\mu_s + p_l \\ (1-\chi_s)[E-p_l+p_m\mu_s] & \frac{(1-\chi_s)}{\chi_s}p_m\mu_s + p_l < E \end{cases} \tag{43}$$

If $p_l \ge \frac{1-\eta_s}{\eta_s}\frac{p_m\mu_s}{\chi_s}$, the leisure function is

$$l(E,p_l,p_m;s) = \begin{cases} \frac{(1-\eta_s)E}{(1-\chi_s\eta_s)p_l} & E \le \frac{1-\eta_s\chi_s}{\eta_s\chi_s}p_m\mu_s \\ (1-\eta_s)\frac{E+p_m\mu_s}{p_l} & \frac{1-\eta_s\chi_s}{\eta_s\chi_s}p_m\mu_s < E < \frac{p_l}{1-\eta_s} - p_m\mu_s \\ 1 & \frac{p_l}{1-\eta_s} - p_m\mu_s \le E \end{cases} \tag{44}$$

If $p_l < \frac{1-\eta_s}{\eta_s}\frac{p_m\mu_s}{\chi_s}$, the leisure function is

$$l(E,p_l,p_m;s) = \begin{cases} \frac{(1-\eta_s)E}{(1-\chi_s\eta_s)p_l} & E < \left[1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right]p_l \\ 1 & \left[1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right]p_l \le E \end{cases} \tag{45}$$

If $p_l \ge \frac{1-\eta_s}{\eta_s}\frac{p_m\mu_s}{\chi_s}$, the remittance function is

$$m(E,p_l,p_m;s) = \begin{cases} 0 & E \le \frac{1-\eta_s\chi_s}{\eta_s\chi_s}p_m\mu_s \\ \frac{\eta_s\chi_s}{p_m}E + (\eta_s\chi_s - 1)\mu_s & \frac{1-\eta_s\chi_s}{\eta_s\chi_s}p_m\mu_s < E < \frac{p_l}{1-\eta_s} - p_m\mu_s \\ \frac{\chi_s}{p_m}(E-p_l) - (1-\chi_s)\mu_s & \frac{p_l}{1-\eta_s} - p_m\mu_s \le E \end{cases} \tag{46}$$

If $p_l < \frac{1-\eta_s}{\eta_s}\frac{p_m\mu_s}{\chi_s}$, the remittance function is

$$m(E,p_l,p_m;s) = \begin{cases} 0 & E \le \frac{(1-\chi_s)}{\chi_s}p_m\mu_s + p_l \\ \frac{\chi_s}{p_m}(E-p_l) - (1-\chi_s)\mu_s & \frac{(1-\chi_s)}{\chi_s}p_m\mu_s + p_l < E \end{cases} \tag{47}$$

For a high-educated household still in school, the consumption function is

$$c_e(E,p_m) = \begin{cases} E & E \le \frac{1-\chi_2}{\chi_2}p_m\mu_2 \\ (1-\chi_2)(E+p_m\mu_2) & E > \frac{1-\chi_2}{\chi_2}p_m\mu_2 \end{cases} \tag{48}$$

and the remittance function is

$$m_e(E,p_m) = \begin{cases} 0 & E \leq \frac{1-\chi_2}{\chi_2} p_m \mu_2 \\ \chi_2 \left(\frac{E}{p_m} + \mu_2\right) - \mu_2 & E > \frac{1-\chi_2}{\chi_2} p_m \mu_2 \end{cases} . \tag{49}$$

Suppose that we let $\mu_0 = 0$, $\chi_0 > 0$, and $\chi_1 = \chi_2 = 0$, so only illegal immigrants care about remittanes and they are necessities, meaning that illegal immigrants will send remittances for any $E > 0$. Notice that since p_m is always multiplied by μ_0 in the consumption and leisure functions (42) and (45)[10], so the price of remittances, p_m, has no effect on an immigrant household's demand for leisure or consumption. Likewise, p_m will have no effect on any of the behavioral functions of a resident household in this case. Meanwhile if $\mu_0 = 0$, the value function V_0 only depends on p_m as a multiplicative factor. Thus if p_m is constant over the lifecycle, it will have no effect on the solution to the intertemporal problem and will not change saving behavior.

On the other hand, if legal residents do care about remittances and $\mu_1, \mu_2 > 0$, p_m does affect the consumption-leisure-remittance bundle in Regime IV where neither the leisure constraint nor the remittance constraint bind. For example, we can see from Eq. (45) that a higher remittance price will cause the household to substitute leisure for remittances and work less. The bigger μ is and the more remittances are a luxury good, the bigger an effect a tax on remittances will have on household behavior, but only in the regimes where the household actually sends remittances.

2.1 Intertemporal Problem

The intertemporal problem has Lagrangian

$$\mathcal{L}(s) = \sum_{t=0}^{2} \left[\beta^t V_s(E_t(s), p_{l,t}(s), p_m) + \lambda_t \left(y_t(s) + R_{eff}(s) b_t(s) - b_{t+1}(s) - E_t(s)\right)\right] + \nu_1 b_1(s) + \nu_2 b_2(s) \tag{50}$$

where $b_0(s) = b_3(s) = 0$. The first-order conditions are

$$\frac{\partial \mathcal{L}(s)}{\partial E_t(s)} = \beta^t \frac{\partial V_s}{\partial E}(E_t(s), p_{l,t}(s), p_m) - \lambda_t = 0 \quad t = 0,1,2$$

$$\frac{\partial \mathcal{L}(s)}{\partial b_t(s)} = \lambda_t R_{eff}(s) - \lambda_{t-1} + \nu_t = 0 \quad t = 1,2,$$

where $\nu_t \geq 0$ with equality if $b_t(s) > 0$. Thus

$$\frac{\partial V_s}{\partial E}(E_t(s), p_{l,t}(s), p_m) \geq \beta R_{eff}(s) \frac{\partial V_s}{\partial E}(E_{t+1}(s), p_{l,t+1}(s), p_m) \quad t = 0,1 \tag{51}$$

with equality if $b_{t+1}(s) > 0$.

3 Calibration

We calibrate the production function as follows. Let $A_{KX}, A_{SX}, A_{XY}, A_{UY} \geq 0$ be productivity indices for capital, high-educated labor, and low-educated labor.

$$X = \left[A_{KX} K^{\sigma_{KS}} + A_{SX} N(2)^{\sigma_{KS}}\right]^{\frac{1}{\sigma_{KS}}} \tag{52}$$

[10] We will always have $p_l \geq \frac{1-\eta_s}{\eta_s} \frac{p_m \mu_s}{\chi_s}$ if $\mu_s = 0$.

χ_{XU}	χ_{KS}	A_{KX}	A_{XY}	δ_{ann}
4.0	0.8	0.5667	0.971466	0.02

Table 1: Calibration of technology parameters.

c_{edu}	G	τ_2/τ_1	τ_e	τ_m	ρ_1	ρ_2
0.000625	0.0015	2	0.0005	0.0	0.044	0.027

Table 2: Calibration of government parameters

is the capital-skill aggregator where $A_{KX} + A_{SX} = 1$, $\sigma_{KS} < 1$ and

$$\xi_{KS} = \frac{1}{1 - \sigma_{KS}} \tag{53}$$

is the elasticity of substitution between capital and skilled labor. The production function is then

$$Y = [A_{XY} X^{\sigma_{XU}} + A_{UY} N(1)^{\sigma_{XU}}]^{\frac{1}{\sigma_{XU}}}, \tag{54}$$

where $A_{XY} + A_{UY} = 1$, $\sigma_{XU} < 1$ and

$$\xi_{XU} = \frac{1}{1 - \sigma_{XU}}. \tag{55}$$

Since $A_{SX} = 1 - A_{KX}$ and $A_{UY} = 1 - A_{XY}$, the production function has two remaining parameters A_{KX} and A_{XY}, which can be calibrated to match the share of capital and the skill premium.

For now, I am using the initial calibration with technology parameters in Table 1, government parameters in Table 2 and preference parameters in Table 3.

The endowment profiles are $e_0(s) = 1$, $e_1(s) = 1.5$, and $e_2(s) = 0.3$ for a low-educated worker with $s = 0, 1$, and $e_1(2) = 1$ and $e_2(2) = 0.75$ for a high-educated worker. In equilibrium,

The income tax on illegal immigrants is

$$\tau(0) = \varepsilon_\tau \tau(1) \tag{56}$$

where $\varepsilon_\tau \in [0, 1]$ is the efficiency of tax collection for illegal immigrants. The replacement rate for illegal immigrants is

$$B(0) = \varepsilon_B B(1), \tag{57}$$

where $\varepsilon_B \in [0, 1]$ is the fairness of Social Security for illegal immigrants.

4 Policy Experiments

The solution algorithm for computing equilibria is described in Appendix C. Equilibrium observables for several policy regimes with the baseline parameters of Tables 1-3 are given in 4. The seventh column

β_{ann}	γ	l_{edu}	η	μ_0	$\mu_1 = \mu_2$	χ_0	$\chi_1 = \chi_2$
0.97	1.0	0.75	0.2	0	0.1	0.2	0.01

Table 3: Calibration of preference parameters.

P	ζ	G	ε_τ	ε_B	$U(1)$	EV	$\%\Delta w(1)$	$\%\Delta w_{at}(1)$	$\frac{N(0)}{n(0)}$	$\frac{N(1)}{n(1)}$	$n(2)$	$\frac{K}{Y}$
1	0.05	0.001500	0	0	−2.6161	−0.0079	−0.07%	−0.79%	0.5913	0.5765	0.0320	2.613
1	0.05	0.001500	0.5	0	−2.6132	0.0000	0.0%	0.0%	0.5908	0.5761	0.0323	2.620
1	0.05	0.001500	1	0	−2.6104	0.0076	0.07%	0.75%	0.5903	0.5758	0.0327	2.626
0.95	0	0.001500	-	-	−2.6200	−0.0173	−0.36%	−1.81%	-	0.5749	0.0293	2.564
0.95	0	0.001429	-	-	−2.6135	0.0009	−0.10%	−0.13%	-	0.5752	0.0304	2.602
0.95	0	0.001427	-	-	−2.6132	0.0000	−0.09%	−0.04%	-	0.5752	0.0304	2.603
0.95	0	0.001425	-	-	−2.6128	−0.0009	−0.08%	0.05%	-	0.5752	0.0305	2.604
1	0.05	0.001506	1	1	−2.6132	−0.0002	−0.09%	−0.06%	0.5796	0.5752	0.0320	2.601
1	0.05	0.001505	1	1	−2.6132	0.0000	−0.09%	−0.04%	0.5796	0.5752	0.0320	2.602
1	0.05	0.001504	1	1	−2.6131	0.0002	−0.09%	−0.02%	0.5796	0.5752	0.0320	2.602
1	0.05	0.001500	1	1	−2.6128	0.0011	−0.07%	0.07%	0.5797	0.5752	0.0321	2.604

Table 4: Macroeconomic observables for several policy regimes.

gives the equivalent variation of the equilibrium relative to the equilibrium with $\varepsilon_\tau = 0.5$ which is probably closest to the factual equilibrium. Likewise, the eighth and ninth columns give the percent change from the equilibrium with $\varepsilon_\tau = 0.5$ in the wage and the after-tax wage

$$w_{at}(s) = (1 - \tau_y(s))w(s) \tag{58}$$

for low-educated legal residents.

In the first three rows, we see the effect of varying the efficiency of tax collection among illegal immigrants. Not surprisingly, utility for legal residents increases with the efficiency of tax collection. Since illegal immigrants do pay some taxes even if they avoid paying income or payroll taxes and many illegal immigrants manage to pass as legal residents, we set $\varepsilon_\tau = 0.5$ as our baseline for welfare comparisons. That illegal immigrants only pay half their share of taxes reduces legal resident welfare by the equivalent of 0.8 percentage points of consumption.

The next fourth rows represent what happens if the illegal immigrants are deported so the population is reduced by 5%. The fourth row shows what happens if the illegal immigrants are deported and government spending, G, on public goods and services remains unchanged. The sixth to eight rows show what happens if the elasticity of G with respect to the population is 0.95, 0.975, and 1.0 respectively. A deportation does not increase legal resident utility unless G decreases by more than $0.975 \times 5\%$.

The last four rows represent what happens if the illegal immigrants are fully legalized. In the last row, G remains unchanged. In the eight to tenth rows, the elasticity of G with respect to the legal population is 0.75, 0.625, and 0.5 respectively. Legalization only increases legal resident utility if G increases by more than $0.625 \times 5\%$.

Note that a good approximation to the equivalent variation, i.e. the percentage by which consumption must be changed under the baseline policy to get the same utility as in the policy under consideration, is the change in after-tax low-educated wages from the baseline policy. A low-educated household's wealth is proportional to the after-tax low-educated wage, so changes in legal resident welfare can mostly be attributed to what happens to the low-educated after-tax wage. Where the policy change has a negligible effect on the after-tax low-educated wage, changes in interest rates may cause legal resident utility and the after-tax low-educated wage to move in opposite directions though.

Contrary to the usual intution that illegal immigrants reduce low-educated wages, in the long run that is not true. Low-educated wages are higher in the first three rows than all the other rows. This can be attributed to the fact that illegal immigrants save more and a higher proportion of legal residents become high educated when illegal immigrants do not get Social Security, as seen in the n_2 column. Why do illegal immigrants incentivize legal residents to invest in higher education? More to the point, why is it that when we fully legalize the immigrants, this incentive disappears? The answer can be seen by comparing $N(0)/n(0)$ to $N(1)/n(1)$, which are both proportional to the average hours worked by the corresponding type of low-educated household. Illegal immigrants work roughly 0.02 more than low-educated legal residents, but this difference mostly disappears when the illegal immigrants are legalized. Thus I conclude that the welfare benefits of having illegal immigrants to legal residents derives primarily from illegal immigrants not getting Social Security benefits. There is also a small effect from illegal immigrants working and saving more because they value remittances more. Interestingly, if we compare the welfare of illegal immigrants under the baseline policy to what they would get if they were legalized, welfare goes down. Immigrants get more benefit from not paying their fair share of taxes than they would get from Social Security.[11]

5 Concluding Remarks

The importance of Social Security for explaining how illegal immigrants affect steady-state welfare suggests that introducing other types of benefits that are provided either by the government or employers, such as health care, would only magnify the effects seen here.

There are some additional frictions that could be readily incorporated into the present model. First, capital market frictions could be implemented so that illegal immigrants cannot save at the same rate of return as legal residents, reflecting the fact that they have less access to investment opportunities. Second, while it would be cumbersome to introduce search frictions into a model where a period is twenty years, as a proxy for search frictions we could introduce a minimum wage that only applies to legal residents, allowing for unemployment among low-educated legal residents.

6 Income-Expenditure Identity

Substituting the budget constraints into Eq. (15), we obtain

$$\begin{aligned} C \;=\; & n(0)\left[w(1)(1-\tau_y(0))(1-l_0(0))e_0(1)-(1+\tau_m)m_0(0)-b_1(0)\right.\\ & +(1-\tau_y(0))w(1)(1-l_1(1))e_1(1)+(\omega R+(1-\omega)(1-\delta))b_1(0)-(1+\tau_m)m_1(0)-b_2(0)\\ & \left.+\,(1-\tau_y(0))w(1)(1-l_2(1))e_2(1)+(\omega R+(1-\omega)(1-\delta))b_2(0)-(1+\tau_m)m_2(0)\right]\\ & +n(1)\left[(1-\tau_y(1))w(1)(1-l_0(1))e_0(1)-(1+\tau_m)m_0(1)-b_1(1)\right.\\ & +(1-\tau_y(1))w(1)(1-l_1(1))e_1(1)+Rb_1(1)-(1+\tau_m)m_1(1)-b_2(1)\\ & \left.+\,(1-\tau_y(1))w(1)(1-l_2(1))e_2(1)+Rb_2(1)+B(1)-(1+\tau_m)m_2(1)\right]\\ & +n(2)\left[-(1+\tau_m)m_0(2)-b_1(2)-\tau_e\right.\\ & +(1-\tau_y(2))w(2)(1-l_1(2))e_1(2)+Rb_1(2)-(1+\tau_m)m_1(2)-b_2(2)\\ & \left.+\,(1-\tau_y(2))w(2)(1-l_2(2))e_2(2)+Rb_2(2)-(1+\tau_m)m_2(2)+B(2)\right] \end{aligned}$$

[11] This ignores any stigma and uncertainty that comes from being an illegal immigrant.

Noting that $e_0(2) = 0$,

$$\begin{aligned}
C &= \sum_{s=0}^{2} n(s)w(s)\sum_{t=0}^{2}(1-l_t(s))e_t(s) \\
&\quad -\sum_{s=0}^{2} n(s)w(s)\tau_y(s)\sum_{t=0}^{2}(1-l_t(s))e_t(s) - (1+\tau_m)\sum_{s=0}^{2} n(s)\sum_{t=0}^{2} m_t(s) \\
&\quad -[\omega+1-\omega]\, n(0)\sum_{t=1}^{2} b_t(0) + \sum_{s=0}^{2} n(s)\sum_{t=1}^{2} b_t(s) + (1-\delta)(1-\omega)n(0)\sum_{t=1}^{2} b_t(0) \\
&\quad +R\left[\omega n(0)\sum_{t=1}^{2} b_t(0) + \sum_{s=1}^{2} n(s)\sum_{t=1}^{2} b_t(s)\right] + \sum_{s=1}^{2} n(s)B(s) - n(2)\tau_e
\end{aligned}$$

$$\begin{aligned}
C &= w(1)N(1) + w(2)N(2) \\
&- \sum_{s=0}^{2} n(s)w(s)\tau_y(s)\sum_{t=0}^{2}(1-l_t(s))e_t(s) - \tau_m M \\
&\quad -M + (R-1)K + \sum_{s=1}^{2} n(s)B(s) - n(2)\tau_e \\
&\quad +(1-\omega)n(0)\,[1-\delta-1]\sum_{t=1}^{2} b_t(0)
\end{aligned}$$

$$\begin{aligned}
C &= w(1)N(1) + w(2)N(2) \\
&\quad -\sum_{s=0}^{2} n(s)w(s)\tau_y(s)\sum_{t=0}^{2}(1-l_t(s))e_t(s) - \tau_m M \\
&\quad -M + (R-1)K + \sum_{s=1}^{2} n(s)B(s) - n(2)\tau_e \\
&\quad -\delta(1-\omega)n(0)\sum_{t=1}^{2} b_t(0)
\end{aligned}$$

Let us define

$$D = (1-\omega)n(0)\sum_{t=1}^{2} b_t(0). \tag{59}$$

$$\begin{aligned}
C &= w(1)N(1) + w(2)N(2) - \left(G + \sum_{s=1}^{2} n(s)B(s)\right) \\
&\quad -M + (R-1)K + \sum_{s=1}^{2} n(s)B(s) - n(2)\tau_e - \delta D.
\end{aligned}$$

Since F has constant returns to scale, we will have the income-product identity:

$$Y = w(1)N(1) + w(2)N(2) + (R-1+\delta)K. \tag{60}$$

Thus

$$C = Y - \delta K - \delta(1-\omega)n(0)\sum_{t=1}^{2} b_t(0) - G - M - \delta D - n(2)\tau_e$$

$$C = Y - \delta K - G - M - n(2)\tau_e.$$

If we define investment as[12]

$$I = \delta(K + D) + n(2)\tau_e,$$

we get

$$Y = C + I + M + G, \tag{61}$$

A Intertemporal Problem

We can write the general problem for an out of school household in recursive form as

$$v_t(x_t; s) = \max_{b_{t+1}, E_t} V_s(E_t, p_l^t, p_m) + \beta v_{t+1}(y_{t+1}(s) + R_{eff}(s)b_{t+1}; s) \tag{62}$$

subject to

$$E_t + b_{t+1} = x_t$$

$$b_{t+1} \geq 0,$$

where the terminal value function is

$$v_2(x_2; s) = V_s(x_2, p_l^2, p_m). \tag{63}$$

The Lagrangian is

$$\mathcal{L}_t(s) = V_s(E_t, p_l^t, p_m) + \beta v_{t+1}(y_{t+1}(s) + R_{eff}(s)b_{t+1}; s) + \lambda_t(s)[x_t - E_t - b_{t+1}] + \mu_{t+1} b_{t+1}. \tag{64}$$

The first-order conditions are

$$\begin{aligned} \frac{\partial \mathcal{L}_t(s)}{\partial E_t} &= \frac{\partial V_s}{\partial E}(E_t, p_l^t, p_m) - \lambda_t(s) = 0 \\ \frac{\partial \mathcal{L}_t(s)}{\partial b_{t+1}} &= \beta R v'_{t+1}(y_{t+1}(s) + R_{eff}(s)b_{t+1}; s) - \lambda_t(s) + \mu_{t+1} = 0. \end{aligned}$$

By the Envelope Theorem,

$$v'_t(x_t; s) = \frac{\partial \mathcal{L}_t(s)}{\partial x_t} = \lambda_t(s) = \frac{\partial V_s}{\partial E}(E_t, p_l^t, p_m).$$

Thus the Euler inequality is

$$\frac{\partial V_s}{\partial E}(E_t, p_l^t, p_m) \geq \beta R_{eff}(s) \frac{\partial V_s}{\partial E}(y_{t+1}(s) + R_{eff}(s)(x_t - E_t), p_m, p_l^{t+1}) \tag{65}$$

[12] Here we include education tuition as part of investment, although it is often classified as consumption in national income and product accounts.

with equality if $b_{t+1} > 0$. If $b_{t+1} = 0$, the Euler equation would be

$$\frac{\partial V_s}{\partial E}(x_t, p_l^t, p_m) \geq \beta R_{eff}(s) \frac{\partial V_s}{\partial E}(y_{t+1}(s), p_l^{t+1}, p_m).$$

We show in Section B that V_E is strictly concave with respect to E. Let us define

$$q(\lambda, p_l, p_m) = \left(\frac{\partial V_s}{\partial E} \right)^{-1} (\lambda, p_l, p_m), \tag{66}$$

which is strictly decreasing. Thus

$$x_t \leq q \left(\beta R \frac{\partial V_s}{\partial E}(y_{t+1}(s), p_l^{t+1}, p_m), p_l^t, p_m \right).$$

Therefore the borrowing constraint will bind for

$$x_t \leq x_t^b = q \left(\beta R \frac{\partial V_s}{\partial E}(y_{t+1}(s), p_l^{t+1}, p_m), p_l^t, p_m \right), \tag{67}$$

in which case

$$E_t(x_t, s) = x_t. \tag{68}$$

Alternatively, if $x_t \geq x_t^b$, the expenditures function $E_t(x_t, s)$ will solve the Euler equation

$$\frac{\partial V_s}{\partial E}(E_t(x_t, s), p_l^t, p_m) = \beta R \frac{\partial V_s}{\partial E}(y_{t+1}(s) + R(x_t - E_t(x_t, s)), p_l^{t+1}, p_m). \tag{69}$$

Note that if the dependence of $\frac{\partial V_s}{\partial E}(E)$ on p_m can be factored out as a proportional function that will cancel out of (69), remittances or taxes on remittances can have no effect on saving at the household level

B Intratemporal Problem

Since g is irrelevant to the intratemporal problem, the Lagrangian for (29) is

$$\mathcal{L} = (c^{1-\chi_s}(m + \mu_s)^{\chi_s})^{\eta_s} l^{1-\eta_s} + \lambda[E - c - p_l l - p_m m] + \zeta(1 - l) + \xi m. \tag{70}$$

Thus the first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial c} = (1 - \chi_s)\eta_s c^{(1-\chi_s)\eta_s - 1}(m + \mu_s)^{\chi_s \eta_s} l^{1-\eta_s} - \lambda = 0 \tag{71}$$

$$\frac{\partial L}{\partial m} = \chi_s \eta_s (m + \mu)^{\chi_s \eta_s - 1} c^{(1-\chi_s)\eta_s} l^{1-\eta_s} - p_m \lambda + \xi = 0 \tag{72}$$

$$\frac{\partial L}{\partial l} = (1 - \eta_s) l^{-\eta_s} c^{(1-\chi_s)\eta_s}(m + \mu_s)^{\chi_s \eta_s} - p_l \lambda - \zeta = 0 \tag{73}$$

Thus we have

$$\lambda = (1 - \chi_s)\eta_s c^{(1-\chi_s)\eta_s - 1}(m + \mu_s)^{\chi_s \eta_s} l^{1-\eta_s}; \tag{74}$$

$$\chi_s \eta_s (m + \mu)^{\chi_s \eta_s - 1} c^{(1-\chi_s)\eta_s} l^{1-\eta_s} \leq p_m (1 - \chi_s)\eta_s c^{(1-\chi_s)\eta_s - 1}(m + \mu_s)^{\chi_s \eta_s} l^{1-\eta_s} \tag{75}$$

with equality if $m > 0$; and

$$(1-\eta_s)l^{-\eta_s}c^{(1-\chi_s)\eta_s}(m+\mu_s)^{\chi_s\eta_s}l^{1-\eta_s} \geq p_l(1-\chi_s)\eta_s c^{(1-\chi_s)\eta_s-1}(m+\mu_s)^{\chi_s\eta_s}l^{1-\eta_s}$$

with equality if $l < 1$. The last two simplify to

$$\frac{\chi_s\eta_s}{m+\mu} \leq \frac{p_m(1-\chi_s)\eta_s}{c} \tag{76}$$

with equality if $m > 0$; and

$$1-\eta_s \geq (1-\chi_s)\eta_s p_l \frac{l}{c} \tag{77}$$

with equality if $l < 1$. That is,

$$c \leq \frac{(1-\chi_s)}{\chi_s} p_m(m+\mu_s) \tag{78}$$

with equality if $m > 0$; and

$$c \geq (1-\chi_s)\frac{\eta_s}{1-\eta_s} p_l l \tag{79}$$

with equality if $l < 1$.

B.1 Regime I: No Work and No Remittances

We will have $l = 1$ and $m = 0$ if

$$c(E, p_l, p_m) = E - p_l. \tag{80}$$

This happens if

$$(1-\chi_s)\frac{\eta_s}{1-\eta_s} p_l \leq c \leq \frac{(1-\chi_s)}{\chi_s} p_m\mu_s$$

$$(1-\chi_s)\frac{\eta_s}{1-\eta_s} p_l \leq c \leq \frac{(1-\chi_s)}{\chi_s} p_m\mu_s$$

$$(1-\chi_s)\frac{\eta_s}{1-\eta_s} p_l \leq E - p_l \leq \frac{(1-\chi_s)}{\chi_s} p_m\mu_s$$

$$\left[1 + (1-\chi_s)\frac{\eta_s}{1-\eta_s}\right] p_l \leq E \leq \frac{(1-\chi_s)}{\chi_s} p_m\mu_s + p_l \tag{81}$$

That can only happen if

$$\frac{\eta_s}{1-\eta_s} p_l < \frac{p_m\mu_s}{\chi_s}$$

$$p_l < \frac{1-\eta_s}{\eta_s}\frac{p_m\mu_s}{\chi_s} \tag{82}$$

In the limit as $\chi_s \to 0$ or $\mu_s \to \infty$, this condition for the existence of the no-work, no-remittance zone will be satisfied.

At the lower threshold,

$$E_{l**} = \left[1 + (1-\chi_s)\frac{\eta_s}{1-\eta_s}\right] p_l. \tag{83}$$

At the upper threshold,

$$E^{m**} = \frac{(1-\chi_s)}{\chi_s} p_m\mu_s + p_l. \tag{84}$$

In the no-work, no-remittance zone, $E_{l**} \leq E \leq E^{m**}$,

$$V_s(E, p_l, p_m) = g(((E - p_l)^{1-\chi_x} {\mu_s}^{\chi_s})^{\eta_s}) \tag{85}$$

$$\frac{\partial V_s}{\partial E}(E, p_l, p_m) = \eta_s(1 - \chi_s) g'(((E - p_l)^{1-\chi_x} {\mu_s}^{\chi_s})^{\eta_s})(E - p_l)^{(1-\chi_s)\eta_s - 1} \mu_s^{\chi_s \eta_s} \tag{86}$$

$$\begin{aligned} \frac{\partial^2 V_s}{\partial E^2}(E, p_l, p_m) &= g''(((E - p_l)^{1-\chi_x} {\mu_s}^{\chi_s})^{\eta_s}) \left[\eta_s(1 - \chi_s)(E - p_l)^{(1-\chi_s)\eta_s - 1} \mu_s^{\chi_s \eta_s}\right]^2 \\ &\quad + [(1 - \chi_s)\eta_s - 1]\eta_s(1 - \chi_s) g'(((E - p_l)^{1-\chi_x} {\mu_s}^{\chi_s})^{\eta_s})(E - p_l)^{(1-\chi_s)\eta_s - 2} \mu_s^{\chi_s \eta_s} \\ &< 0 \end{aligned}$$

since g is strictly concave and $(1 - \chi_s)\eta_s \leq 1$.

B.2 Regime II: Work and No Remittances

Now suppose that $l < 1$ and $m = 0$. Then we must have

$$c = (1 - \chi_s)\frac{\eta_s}{1 - \eta_s} p_l l \leq \frac{(1 - \chi_s)}{\chi_s} p_m \mu_s. \tag{87}$$

$$c + p_l l = E$$

$$\left[p_l + (1 - \chi_s)\frac{\eta_s}{1 - \eta_s} p_l\right] l = E$$

$$l\,(E, p_l, p_m) = \frac{E}{\left(1 + (1 - \chi_s)\frac{\eta_s}{1-\eta_s}\right) p_l}$$

$$l\,(E, p_l, p_m) = \frac{(1 - \eta_s)E}{(1 - \chi_s \eta_s)\, p_l} \tag{88}$$

$$c(E, p_l, p_m) = \frac{(1 - \chi_s)\frac{\eta_s}{1-\eta_s}}{1 + (1 - \chi_s)\frac{\eta_s}{1-\eta_s}} E$$

$$c(E, p_l, p_m) = \frac{(1 - \chi_s)\eta_s}{1 - \chi_s \eta_s} E \tag{89}$$

By assumption we must have

$$\frac{E}{\left(1 + (1 - \chi_s)\frac{\eta_s}{1-\eta_s}\right) p_l} < 1,$$

so

$$E < \left(1 + (1 - \chi_s)\frac{\eta_s}{1 - \eta_s}\right) p_l = E_{l**}.$$

But we must also have

$$\frac{(1 - \chi_s)\frac{\eta_s}{1-\eta_s}}{1 + (1 - \chi_s)\frac{\eta_s}{1-\eta_s}} E \leq \frac{(1 - \chi_s)}{\chi_s} p_m \mu_s$$

$$E \leq \frac{1 + (1 - \chi_s)\frac{\eta_s}{1-\eta_s}}{\frac{\eta_s}{1-\eta_s}} \frac{p_m \mu_s}{\chi_s}$$

$$E \leq \left[\frac{1-\eta_s}{\eta_s} + 1 - \chi_s\right] \frac{p_m \mu_s}{\chi_s}.$$

Define this upper threshold to be

$$\begin{aligned} E^{m*} &= \left[\frac{1-\eta_s}{\eta_s} + 1 - \chi_s\right] \frac{p_m \mu_s}{\chi_s} \\ &= \frac{1-\eta_s+\eta_s-\eta_s\chi_s}{\eta_s} \frac{p_m \mu_s}{\chi_s} \end{aligned}$$

$$E^{m*} = \frac{1-\eta_s\chi_s}{\eta_s\chi_s} p_m \mu_s \tag{90}$$

Let us define the upper threshold to be

$$E^* = \min\left\{E^{m*}, E_{l**}\right\}. \tag{91}$$

If (82) holds,

$$E_{l**} = \left[1 + (1-\chi_s)\frac{\eta_s}{1-\eta_s}\right] p_l < \left[1 + (1-\chi_s)\frac{\eta_s}{1-\eta_s}\right] \frac{1-\eta_s}{\eta_s} \frac{p_m \mu_s}{\chi_s} = \left[\frac{1-\eta_s}{\eta_s} + 1 - \chi_s\right] \frac{p_m \mu_s}{\chi_s} = E^{m*}$$

so (82) is the condition that $E^* = E_{l**}$. If (82) holds, $E^* = E_{l**}$ and for E slightly higher than E^*, the household will stop working but still not pay remittances. If (82) holds, $E^* = E^{m**}$ and for E slightly higher than E^*, the household will still work but pay remittances.

For $E < E^*$,

$$V_s(E, p_l, p_m) = g\left(\mu_s^{\chi_s\eta_s}\left(\frac{(1-\chi_s)\eta_s}{1-\chi_s\eta_s}E\right)^{(1-\chi_s)\eta_s}\left(\frac{(1-\eta_s)E}{(1-\chi_s\eta_s)\,p_l}\right)^{1-\eta_s}\right)$$

$$V_s(E, p_l, p_m) = g\left(\mu_s^{\chi_s\eta_s}\left(\frac{(1-\chi_s)\eta_s}{1-\chi_s\eta_s}E\right)^{\eta_s-\chi_s\eta_s}\left(\frac{(1-\eta_s)E}{(1-\chi_s\eta_s)\,p_l}\right)^{1-\eta_s}\right)$$

$$V_s(E, p_l, p_m) = g\left(\mu_s^{\chi_s\eta_s}\left(\frac{(1-\chi_s)\eta_s}{1-\chi_s\eta_s}E\right)^{\eta_s-\chi_s\eta_s}\left(\frac{(1-\eta_s)E}{(1-\chi_s\eta_s)\,p_l}\right)^{1-\eta_s}\right)$$

$$V_s(E, p_l, p_m) = g\left(\mu_s^{\chi_s\eta_s}\left((1-\chi_s)\,\eta_s\right)^{\eta_s(1-\chi_s)}\left(\frac{1-\eta_s}{p_l}\right)^{1-\eta_s}\left(\frac{E}{1-\chi_s\eta_s}\right)^{1-\chi_s\eta_s}\right) \tag{92}$$

Thus

$$\begin{aligned} \frac{\partial V_s}{\partial E}(E, p_l, p_m) &= g'\left(\mu_s^{\chi_s\eta_s}\frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}}{p_l^{1-\eta_s}}\left(\frac{E}{\left(1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)}\right)^{1-\chi_s\eta_s}\right) \\ &\quad \times (1-\chi_s\eta_s)\mu_s^{\chi_s\eta_s}\frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}}{p_l^{1-\eta_s}}\left(\frac{1}{\left(1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)}\right)^{1-\chi_s\eta_s} E^{-\chi_s\eta_s} \end{aligned} \tag{93}$$

$$\frac{\partial^2 V_s}{\partial E^2}(E,p_l,p_m) = g''\left(\mu_s^{\chi_s\eta_s}\frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}}{p_l^{1-\eta_s}}\left(\frac{E}{\left(1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)}\right)^{1-\chi_s\eta_s}\right)$$

$$\times\left[(1-\chi_s\eta_s)\mu_s^{\chi_s\eta_s}\frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}}{p_l^{1-\eta_s}}\left(\frac{1}{\left(1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)}\right)^{1-\chi_s\eta_s}E^{-\chi_s\eta_s}\right]^2$$

$$-g'\left(\mu_s^{\chi_s\eta_s}\frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}}{p_l^{1-\eta_s}}\left(\frac{E}{\left(1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)}\right)^{1-\chi_s\eta_s}\right)$$

$$\times\chi_s\eta_s(1-\chi_s\eta_s)\mu_s^{\chi_s\eta_s}\frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}}{p_l^{1-\eta_s}}\left(\frac{1}{\left(1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)}\right)^{1-\chi_s\eta_s}E^{-\chi_s\eta_s} \quad (94)$$

$$< 0$$

since $\chi_s, \eta_s \in [0,1]$.

Suppose (82) holds,

$$\lim_{E\downarrow E_{l**}}\frac{\partial V_s}{\partial E}(E,p_l,p_m) = \eta_s(1-\chi_s)g'(((E_{**}-p_l)^{1-\chi_x}\mu_s{}^{\chi_s})^{\eta_s})(E_{l**}-p_l)^{(1-\chi_s)\eta_s-1}\mu_s^{\chi_s\eta_s} \quad (95)$$

$$\lim_{E\uparrow E_{l**}}\frac{\partial V_s}{\partial E}(E,p_l,p_m) = g'\left(\mu_s^{\chi_s\eta_s}\frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}}{p_l^{1-\eta_s}}\left(\frac{E_{l**}}{\left(1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)}\right)^{1-\chi_s\eta_s}\right)$$

$$\times(1-\chi_s\eta_s)\mu_s^{\chi_s\eta_s}\frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}}{p_l^{1-\eta_s}}\left(\frac{1}{\left(1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)}\right)^{1-\chi_s\eta_s}E_{l**}^{-\chi_s\eta_s} \quad (96)$$

If $\partial V_s/\partial E$ is continuous at E_{l**}, we must have the arguments of g' in (95)-(96) must be the same.

$$E_{l**} - p_l = (1-\chi_s)\frac{\eta_s}{1-\eta_s}p_l$$

The argument in (95) is

$$(E_{l**}-p_l)^{\eta_s(1-\chi_s)} = \left[(1-\chi_s)\frac{\eta_s}{1-\eta_s}p_l\right]^{\eta_s(1-\chi_s)}.$$

The argument in (96) is

$$\frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}}{p_l^{1-\eta_s}}\left(\frac{E_{l**}}{\left(1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)}\right)^{1-\chi_s\eta_s} = \frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}}{p_l^{1-\eta_s}}\left(\frac{\left[1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right]p_l}{\left(1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)}\right)^{1-\chi_s\eta_s}$$

$$= \left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}p_l^{\eta_s(1-\chi_s)},$$

so they are the same. The factors outside g' must also be the same. The factor in (95) is

$$\eta_s(1-\chi_s)(E_{l**}-p_l)^{(1-\chi_s)\eta_s-1} = \eta_s(1-\chi_s)\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}p_l\right)^{(1-\chi_s)\eta_s-1}.$$

The factor in (96) is

$$\begin{aligned}
& (1-\chi_s\eta_s)\frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}}{p_l^{1-\eta_s}}\left(\frac{1}{\left(1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)}\right)^{1-\chi_s\eta_s} E_{l**}^{-\chi_s\eta_s} \\
= & (1-\chi_s\eta_s)\frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}}{p_l^{1-\eta_s}}\left(\frac{1}{\left(1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)}\right)^{1-\chi_s\eta_s} \left(\left[1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right]p_l\right)^{-\chi_s\eta_s} \\
= & (1-\chi_s\eta_s)\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}\left(1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{-1} p_l^{-1+\eta_s-\chi_s\eta_s} \\
= & (1-\chi_s\eta_s)\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s-1+1}\left(1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right) p_l^{(1-\chi_s)\eta_s-1} \\
= & (1-\chi_s\eta_s)\frac{(1-\chi_s)\frac{\eta_s}{1-\eta_s}}{1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}}\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}p_l\right)^{(1-\chi_s)\eta_s-1} \\
= & (1-\chi_s\eta_s)\frac{(1-\chi_s)\eta_s}{1-\eta_s+(1-\chi_s)\eta_s}\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}p_l\right)^{(1-\chi_s)\eta_s-1} \\
= & (1-\chi_s\eta_s)\frac{(1-\chi_s)\eta_s}{1-\chi_s\eta_s}\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}p_l\right)^{(1-\chi_s)\eta_s-1} \\
= & (1-\chi_s)\eta_s\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}p_l\right)^{(1-\chi_s)\eta_s-1}
\end{aligned}$$

so, indeed, they are the same. $\partial V/\partial E$ is continuous at E_{l**} if (82) holds.

B.3 Regime III: Remittances and No Work

Now suppose that $m > 0$ and $l = 1$. This will happen if

$$c = \frac{(1-\chi_s)}{\chi_s}p_m(m+\mu_s) \geq (1-\chi_s)\frac{\eta_s}{1-\eta_s}p_l. \tag{97}$$

Since

$$c + p_m m + p_l = E,$$

)we have

$$\left[\frac{(1-\chi_s)}{\chi_s}(m+\mu_s)+m\right]p_m = E - p_l$$

$$\frac{(1-\chi_s)\mu_s + m}{\chi_s}p_m = E - p_l$$

$$(1-\chi_s)\mu_s + m = \frac{\chi_s}{p_m}(E-p_l)$$

$$m(E, p_l, p_m) = \frac{\chi_s}{p_m}(E - p_l) - (1 - \chi_s)\mu_s \tag{98}$$

$$\begin{aligned} c &= E - p_l - p_m m \\ &= E - p_l - \chi_s(E - p_l) + (1 - \chi_s)p_m\mu_s \end{aligned}$$

$$c(E, p_l, p_m) = (1 - \chi_s)[E - p_l + p_m\mu_s] \tag{99}$$

$$\begin{aligned} \frac{1-\chi_s}{\chi_s} p_m \left[\frac{\chi_s}{p_m}(E - p_l) - (1 - \chi_s)\mu_s + \mu_s\right] &= \frac{1-\chi_s}{\chi_s} p_m \left[\frac{\chi_s}{p_m}(E - p_l) + \chi_s\mu_s\right] \\ &= (1 - \chi_s)[E - p_l + p_m\mu_s] \end{aligned}$$

so that checks. This must satisfy the inequality in (97), so we have

$$(1 - \chi_s)[E - p_l + p_m\mu_s] \geq (1 - \chi_s)\frac{\eta_s}{1 - \eta_s}p_l$$

$$E - p_l + p_m\mu_s \geq \frac{\eta_s}{1 - \eta_s}p_l$$

$$E \geq \left(1 + \frac{\eta_s}{1 - \eta_s}\right)p_l - p_m\mu_s = \frac{p_l}{1 - \eta_s} - p_m\mu_s$$

Let us define

$$E_{l*} = \frac{p_l}{1 - \eta_s} - p_m\mu_s \tag{100}$$

to be this lower bound. We must also satisfy the condition that $m > 0$, so

$$\frac{\chi_s}{p_m}(E - p_l) - (1 - \chi_s)\mu_s > 0$$

$$E - p_l > \frac{1 - \chi_s}{\chi_s}p_m\mu_s$$

$$E > p_l + \frac{1 - \chi_s}{\chi_s}p_m\mu_s = E^{m**}.$$

Let us define the lower threshold to be

$$E_* = \max\{E_{l*}, E^{m**}\}. \tag{101}$$

The condition $E_{l*} < E^{m**}$ is

$$\frac{p_l}{1 - \eta_s} - p_m\mu_s < p_l + \frac{1 - \chi_s}{\chi_s}p_m\mu_s$$

$$\frac{p_l}{1 - \eta_s} - \frac{1 - \eta_s}{1 - \eta_s}p_l < \frac{p_m\mu_s}{\chi_s}$$

$$\frac{\eta_s}{1 - \eta_s}p_l < \frac{p_m\mu_s}{\chi_s}$$

$$p_l < \frac{1 - \eta_s}{\eta_s}\frac{p_m\mu_s}{\chi_s},$$

which is the same as (82). Thus that is the condition such that for E slightly less than E_* that remittances will be zero while the household still does not work. If (82) does not hold, for E slightly less than E_* the household will start working and pay remittances.

For $E > E_*$,

$$m + \mu_s = \frac{\chi_s}{p_m}(E - p_l) - (1-\chi_s)\mu_s + \mu_s$$

$$m + \mu_s = \frac{\chi_s}{p_m}\left[E - p_l + p_m\mu_s\right]. \tag{102}$$

$$V_s(E,p_l,p_m) = g\left(\left(\frac{\chi_s}{p_m}\left[E - p_l + p_m\mu_s\right]\right)^{\chi_s\eta_s}\left((1-\chi_s)[E - p_l + p_m\mu_s]\right)^{(1-\chi_s)\eta_s}\right)$$

$$V_s(E,p_l,p_m) = g\left(\left(\frac{\chi_s}{p_m}\right)^{\chi_s\eta_s}(1-\chi_s)^{(1-\chi_s)\eta_s}\left[E - p_l + p_m\mu_s\right]^{\eta_s}\right). \tag{103}$$

$$\frac{\partial V_s}{\partial E} = g'\left(\left(\frac{\chi_s}{p_m}\right)^{\chi_s\eta_s}(1-\chi_s)^{(1-\chi_s)\eta_s}\left[E - p_l + p_m\mu_s\right]^{\eta_s}\right)\eta_s\left(\frac{\chi_s}{p_m}\right)^{\chi_s\eta_s}(1-\chi_s)^{(1-\chi_s)\eta_s}\left[E - p_l + p_m\mu_s\right]^{\eta_s-1} \tag{104}$$

$$\begin{aligned}\frac{\partial^2 V_s}{\partial E^2} &= g''\left(\left(\frac{\chi_s}{p_m}\right)^{\chi_s\eta_s}(1-\chi_s)^{(1-\chi_s)\eta_s}\left[E - p_l + p_m\mu_s\right]^{\eta_s}\right)\left(\eta_s\left(\frac{\chi_s}{p_m}\right)^{\chi_s\eta_s}(1-\chi_s)^{(1-\chi_s)\eta_s}\left[E - p_l + p_m\mu_s\right]^{\eta_s-1}\right)^2\\ &\quad -(1-\eta_s)g'\left(\left(\frac{\chi_s}{p_m}\right)^{\chi_s\eta_s}(1-\chi_s)^{(1-\chi_s)\eta_s}\left[E - p_l + p_m\mu_s\right]^{\eta_s}\right)\eta_s\left(\frac{\chi_s}{p_m}\right)^{\chi_s\eta_s}(1-\chi_s)^{(1-\chi_s)\eta_s}\left[E - p_l + p_m\mu_s\right]^{\eta_s-2}\\ &< 0.\end{aligned}$$

Thus V_s is strictly concave in Regime III.

Suppose that (82) holds.

$$\lim_{E\uparrow E^{m**}}\frac{\partial V_s}{\partial E} = \eta_s(1-\chi_s)g'(((E^{m**} - p_l)^{1-\chi_x}{\mu_s}^{\chi_s})^{\eta_s})(E^{m**} - p_l)^{(1-\chi_s)\eta_s-1}\mu_s^{\chi_s\eta_s}$$

$$E^{m**} - p_l = \frac{(1-\chi_s)}{\chi_s}p_m\mu_s$$

$$\begin{aligned}\lim_{E\uparrow E^{m**}}\frac{\partial V_s}{\partial E} &= g'\left(\left(\frac{(1-\chi_s)}{\chi_s}p_m\mu_s\right)^{(1-\chi_x)\eta_s}{\mu_s}^{\chi_s\eta_s}\right)\left(\frac{(1-\chi_s)}{\chi_s}p_m\mu_s\right)^{(1-\chi_s)\eta_s-1}\mu_s^{\chi_s\eta_s}\\ &= g'\left(\left(\frac{(1-\chi_s)}{\chi_s}p_m\right)^{(1-\chi_x)\eta_s}{\mu_s}^{\eta_s}\right)\eta_s(1-\chi_s)\left(\frac{1-\chi_s}{\chi_s}p_m\right)^{(1-\chi_s)\eta_s-1}\mu_s^{\eta_s-1}\end{aligned}$$

$$\begin{aligned}\lim_{E\downarrow E^{m**}}\frac{\partial V_s}{\partial E} &= g'\left(\left(\frac{\chi_s}{p_m}\right)^{\chi_s\eta_s}(1-\chi_s)^{(1-\chi_s)\eta_s}\left[E^{m**} - p_l + p_m\mu_s\right]^{\eta_s}\right)\\ &\quad\times\eta_s\left(\frac{\chi_s}{p_m}\right)^{\chi_s\eta_s}(1-\chi_s)^{(1-\chi_s)\eta_s}\left[E^{m**} - p_l + p_m\mu_s\right]^{\eta_s-1}\end{aligned}$$

$$E^{m**} - p_l + p_m\mu_s = \frac{(1-\chi_s)}{\chi_s}p_m\mu_s + p_m\mu_s = \frac{p_m\mu_s}{\chi_s}$$

$$\lim_{E \downarrow E^{m**}} \frac{\partial V_s}{\partial E} = g'\left(\left(\frac{\chi_s}{p_m}\right)^{\chi_s \eta_s} (1-\chi_s)^{(1-\chi_s)\eta_s} \left(\frac{p_m \mu_s}{\chi_s}\right)^{\eta_s}\right)$$
$$\times \eta_s \left(\frac{\chi_s}{p_m}\right)^{\chi_s \eta_s} (1-\chi_s)^{(1-\chi_s)\eta_s} \left(\frac{p_m \mu_s}{\chi_s}\right)^{\eta_s - 1}$$

$$\lim_{E \downarrow E^{m**}} \frac{\partial V_s}{\partial E} = g'\left(\left(\frac{1-\chi_s}{\chi_s} p_m\right)^{(1-\chi_s)\eta_s} \mu_s^{\eta_s}\right) \eta_s (1-\chi_s) \left(\frac{1-\chi_s}{\chi_s} p_m\right)^{(1-\chi_s)\eta_s - 1} \mu_s^{\eta_s - 1} = \lim_{E \uparrow E^{m**}} \frac{\partial V_s}{\partial E}.$$

Thus $\partial V_s / \partial E$ is continuous at E^{m**}.

B.4 Regime IV: Work and Remittances

Finally, suppose $l < 1$ and $m > 0$. Then

$$c = \frac{(1-\chi_s)}{\chi_s} p_m (m + \mu_s) = (1-\chi_s) \frac{\eta_s}{1-\eta_s} p_l l. \tag{105}$$

Then

$$m = \frac{\chi_s}{1-\chi_s} \frac{c}{p_m} - \mu_s$$
$$l = \frac{1}{1-\chi_s} \frac{1-\eta_s}{\eta_s} \frac{c}{p_l}$$
$$c + p_l l + p_m m = E$$
$$c + \frac{1}{1-\chi_s} \frac{1-\eta_s}{\eta_s} c + \frac{\chi_s}{1-\chi_s} c - p_m \mu_s = E$$
$$\left[1 + \frac{1}{1-\chi_s}\left(\chi_s + \frac{1-\eta_s}{\eta_s}\right)\right] c = E + p_m \mu_s$$
$$c = \frac{E + p_m \mu_s}{1 + \frac{1}{1-\chi_s}\left(\chi_s + \frac{1-\eta_s}{\eta_s}\right)}$$
$$c = (1-\chi_s) \frac{E + p_m \mu_s}{1 - \chi_s + \chi_s + \frac{1-\eta_s}{\eta_s}}$$
$$c = (1-\chi_s) \frac{E + p_m \mu_s}{\frac{\eta_s}{\eta_s} + \frac{1-\eta_s}{\eta_s}}$$
$$c = (1-\chi_s) \frac{E + p_m \mu_s}{\frac{1}{\eta_s}}$$
$$c(E, p_l, p_m) = \eta_s (1-\chi_s)[E + p_m \mu_s] \tag{106}$$
$$l = \frac{1}{1-\chi_s} \frac{1-\eta_s}{\eta_s} \frac{1}{p_l} \eta_s (1-\chi_s)[E + p_m \mu_s]$$
$$l(E, p_l, p_m) = (1-\eta_s) \frac{E + p_m \mu_s}{p_l} \tag{107}$$
$$m + \mu_s = \frac{\chi_s}{1-\chi_s} \frac{1}{p_m} \eta_s (1-\chi_s)[E + p_m \mu_s]$$

$$m + \mu_s = \eta_s \chi_s \frac{E + p_m \mu_s}{p_m} \tag{108}$$

$$m = \eta_s \chi_s \frac{E + p_m \mu_s}{p_m} - \mu_s$$

$$m(E, p_l, p_m) = \frac{\eta_s \chi_s}{p_m} E + (\eta_s \chi_s - 1)\mu_s \tag{109}$$

Since the household is working,

$$(1 - \eta_s)\frac{E + p_m \mu_s}{p_l} < 1$$

$$E + p_m \mu_s < \frac{p_l}{1 - \eta_s}$$

$$E < \frac{p_l}{1 - \eta_s} - p_m \mu_s = E_{l*}.$$

Since remittances are positive,

$$\frac{\eta_s \chi_s}{p_m} E + (\eta_s \chi_s - 1)\mu_s > 0$$

$$E > \frac{1 - \eta_s \chi_s}{\eta_s \chi_s} p_m \mu_s = E^{m*}.$$

Thus Regime IV is (E^{m*}, E_{l*}). The condition $E^{m*} \le E_{l*}$ is

$$\frac{1 - \eta_s \chi_s}{\eta_s \chi_s} p_m \mu_s \le \frac{p_l}{1 - \eta_s} - p_m \mu_s$$

$$\frac{p_m \mu_s}{\eta_s \chi_s} \le \frac{p_l}{1 - \eta_s}$$

$$\frac{1 - \eta_s}{\eta_s} \frac{p_m \mu_s}{\chi_s} \le p_l,$$

which is the opposite of (82). Thus if (82) holds, $\mathbf{R}_{++}$ decomposes into $(0, E_{l**}) \cup [E_{l**}, E^{m**}] \cup (E^{m**}, \infty)$, where $(0, E_{l**})$ is in Regime II, $[E_{l**}, E^{m**}]$ is in Regime I, and (E^{m**}, ∞) is in Regime III. If (82) does not hold, $\mathbf{R}_{++}$ decomposes into $(0, E^{m*}] \cup (E^{m*}, E_{l*}) \cup [E_{l*}, \infty)$, where $(0, E^{m*}]$ is in Regime II, (E^{m*}, E_{l*}) is in Regime IV, and $[E_{l*}, \infty)$ is in Regime III.

For $E \in (E^{m*}, E_{l*})$, indirect utility is

$$\begin{aligned} V_s(E, p_l, p_m) &= g\left((c^{1-\chi_s}(m + \mu_s)^{\chi_s})^{\eta_s} l^{1-\eta_s}\right) \\ &= g\left(\left(\eta_s(1 - \chi_s)\left(E + p_m \mu_s\right)\right)^{\eta_s - \eta_s \chi_s} \left(\eta_s \chi_s \frac{E + p_m \mu_s}{p_m}\right)^{\eta_s \chi_s} \left((1 - \eta_s)\frac{E + p_m \mu_s}{p_l}\right)^{1-\eta_s}\right) \end{aligned}$$

$$V_s(E, p_l, p_m) = g\left(\left(\eta_s(1 - \chi_s)\right)^{\eta_s(1-\chi_s)} \left(\frac{\eta_s \chi_s}{p_m}\right)^{\eta_s \chi_s} \left(\frac{1 - \eta_s}{p_l}\right)^{1-\eta_s} (E + p_m \mu_s)\right) \tag{110}$$

$$\begin{aligned} \frac{\partial V_s}{\partial E}(E, p_l, p_m) &= \left(\eta_s(1 - \chi_s)\right)^{\eta_s(1-\chi_s)} \left(\frac{\eta_s \chi_s}{p_m}\right)^{\eta_s \chi_s} \left(\frac{1 - \eta_s}{p_l}\right)^{1-\eta_s} \\ &\quad \times g'\left(\left(\eta_s(1 - \chi_s)\right)^{\eta_s(1-\chi_s)} \left(\frac{\eta_s \chi_s}{p_m}\right)^{\eta_s \chi_s} \left(\frac{1 - \eta_s}{p_l}\right)^{1-\eta_s} (E + p_m \mu_s)\right) \end{aligned} \tag{111}$$

$$\begin{aligned}\frac{\partial^2 V_s}{\partial E^2}(E,p_l,p_m) &= \left((\eta_s(1-\chi_s))^{\eta_s(1-\chi_s)} \left(\frac{\eta_s\chi_s}{p_m}\right)^{\eta_s\chi_s} \left(\frac{1-\eta_s}{p_l}\right)^{1-\eta_s} \right)^2 \\ &\quad \times g''\left((\eta_s(1-\chi_s))^{\eta_s(1-\chi_s)} \left(\frac{\eta_s\chi_s}{p_m}\right)^{\eta_s\chi_s} \left(\frac{1-\eta_s}{p_l}\right)^{1-\eta_s} (E+p_m\mu_s) \right) \\ &< 0\end{aligned}$$

Thus V_s is strictly concave in Regime IV. Therefore V_s is strictly concave everywhere.

$$\begin{aligned}\lim_{E\uparrow E^{m*}} \frac{\partial V_s}{\partial E} &= g'\left(\mu_s^{\chi_s\eta_s} \frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}}{p_l^{1-\eta_s}} \left(\frac{E^{m*}}{\left(1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)} \right)^{1-\chi_s\eta_s} \right) \\ &\quad \times(1-\chi_s\eta_s)\mu_s^{\chi_s\eta_s} \frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}}{p_l^{1-\eta_s}} \left(\frac{1}{\left(1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)} \right)^{1-\chi_s\eta_s} (E^{m*})^{-\chi_s\eta_s}\end{aligned}$$

$$\begin{aligned}\lim_{E\uparrow E^{m*}} \frac{\partial V_s}{\partial E} &= g'\left(\mu_s^{\chi_s\eta_s} \frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}}{p_l^{1-\eta_s}} \left(\frac{(1-\eta_s)E^{m*}}{1-\eta_s+(1-\chi_s)\eta_s} \right)^{1-\chi_s\eta_s} \right) \\ &\quad \times(1-\chi_s\eta_s)\mu_s^{\chi_s\eta_s} \frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}}{p_l^{1-\eta_s}} \left(\frac{1-\eta_s}{1-\eta_s+(1-\chi_s)\eta_s} \right)^{1-\chi_s\eta_s} (E^{m*})^{-\chi_s\eta_s}\end{aligned}$$

$$\begin{aligned}\lim_{E\uparrow E^{m*}} \frac{\partial V_s}{\partial E} &= g'\left(\mu_s^{\chi_s\eta_s} \frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}}{p_l^{1-\eta_s}} \left(\frac{(1-\eta_s)E^{m*}}{1-\chi_s\eta_s} \right)^{1-\chi_s\eta_s} \right) \\ &\quad \times(1-\chi_s\eta_s)\mu_s^{\chi_s\eta_s} \frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}}{p_l^{1-\eta_s}} \left(\frac{1-\eta_s}{1-\chi_s\eta_s} \right)^{1-\chi_s\eta_s} (E^{m*})^{-\chi_s\eta_s}\end{aligned}$$

$$\begin{aligned}\lim_{E\uparrow E^{m*}} \frac{\partial V_s}{\partial E} &= g'\left(\mu_s^{\chi_s\eta_s} \frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}}{p_l^{1-\eta_s}} \left(\frac{(1-\eta_s)}{1-\chi_s\eta_s} \frac{1-\eta_s\chi_s}{\eta_s\chi_s} p_m\mu_s \right)^{1-\chi_s\eta_s} \right) \\ &\quad \times(1-\chi_s\eta_s)\mu_s^{\chi_s\eta_s} \frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{(1-\chi_s)\eta_s}}{p_l^{1-\eta_s}} \left(\frac{1-\eta_s}{1-\chi_s\eta_s} \right)^{1-\chi_s\eta_s} \left(\frac{1-\eta_s\chi_s}{\eta_s\chi_s} p_m\mu_s \right)^{-\chi_s\eta_s}\end{aligned}$$

$$\begin{aligned}\lim_{E\uparrow E^{m*}} \frac{\partial V_s}{\partial E} &= g'\left(\mu_s^{\chi_s\eta_s} \frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{\eta_s-\chi_s\eta_s}}{p_l^{1-\eta_s}} \left(\frac{1-\eta_s}{\eta_s\chi_s} p_m\mu_s \right)^{1-\chi_s\eta_s} \right) \\ &\quad \times\mu_s^{\chi_s\eta_s} \frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{\eta_s-\chi_s\eta_s}}{p_l^{1-\eta_s}} (1-\eta_s)^{1-\chi_s\eta_s} \left(\frac{p_m\mu_s}{\eta_s\chi_s} \right)^{-\chi_s\eta_s}\end{aligned}$$

$$\lim_{E\uparrow E^{m*}}\frac{\partial V_s}{\partial E} = g'\left(\frac{(1-\chi_s)^{\eta_s-\chi_s\eta_s}}{p_l^{1-\eta_s}}\left(\frac{1-\eta_s}{\eta_s}\right)^{1-\eta_s}\left(\frac{p_m}{\chi_s}\right)^{1-\chi_s\eta_s}\mu_s\right)$$
$$\times\eta_s\frac{\left((1-\chi_s)\frac{\eta_s}{1-\eta_s}\right)^{\eta_s-\chi_s\eta_s}}{p_l^{1-\eta_s}}\left(\frac{1-\eta_s}{\eta_s}\right)^{1-\chi_s\eta_s}\left(\frac{p_m}{\chi_s}\right)^{-\chi_s\eta_s}$$

$$\lim_{E\uparrow E^{m*}}\frac{\partial V_s}{\partial E} = g'\left((1-\chi_s)^{\eta_s-\chi_s\eta_s}\left(\frac{1-\eta_s}{\eta_s p_l}\right)^{1-\eta_s}\left(\frac{p_m}{\chi_s}\right)^{1-\chi_s\eta_s}\mu_s\right)\eta_s(1-\chi_s)^{\eta_s-\chi_s\eta_s}\left(\frac{1-\eta_s}{\eta_s p_l}\right)^{1-\eta_s}\left(\frac{p_m}{\chi_s}\right)^{-\chi_s\eta_s}$$

$$\lim_{E\downarrow E^{m*}}\frac{\partial V_s}{\partial E} = (\eta_s(1-\chi_s))^{\eta_s(1-\chi_s)}\left(\frac{\eta_s\chi_s}{p_m}\right)^{\eta_s\chi_s}\left(\frac{1-\eta_s}{p_l}\right)^{1-\eta_s}$$
$$\times g'\left((\eta_s(1-\chi_s))^{\eta_s(1-\chi_s)}\left(\frac{\eta_s\chi_s}{p_m}\right)^{\eta_s\chi_s}\left(\frac{1-\eta_s}{p_l}\right)^{1-\eta_s}(E^{m*}+p_m\mu_s)\right)$$

$$E^{\mu*}+p_m\mu_s = \frac{1-\eta_s\chi_s}{\eta_s\chi_s}p_m\mu_s + p_m\mu_s = \frac{p_m\mu_s}{\eta_s\chi_s}$$

$$\lim_{E\downarrow E^{m*}}\frac{\partial V_s}{\partial E} = (\eta_s(1-\chi_s))^{\eta_s(1-\chi_s)}\left(\frac{\eta_s\chi_s}{p_m}\right)^{\eta_s\chi_s}\left(\frac{1-\eta_s}{p_l}\right)^{1-\eta_s}$$
$$\times g'\left((\eta_s(1-\chi_s))^{\eta_s(1-\chi_s)}\left(\frac{\eta_s\chi_s}{p_m}\right)^{\eta_s\chi_s}\left(\frac{1-\eta_s}{p_l}\right)^{1-\eta_s}\frac{p_m\mu_s}{\eta_s\chi_s}\right)$$

$$\lim_{E\downarrow E^{m*}}\frac{\partial V_s}{\partial E} = \eta_s^{\eta_s}(1-\chi_s)^{\eta_s(1-\chi_s)}\left(\frac{\chi_s}{p_m}\right)^{\eta_s\chi_s}\left(\frac{1-\eta_s}{p_l}\right)^{1-\eta_s}$$
$$\times g'\left((\eta_s(1-\chi_s))^{\eta_s(1-\chi_s)}\left(\frac{p_m}{\eta_s\chi_s}\right)^{1-\eta_s\chi_s}\left(\frac{1-\eta_s}{p_l}\right)^{1-\eta_s}\mu_s\right)$$

$$\lim_{E\downarrow E^{m*}}\frac{\partial V_s}{\partial E} = \eta_s(1-\chi_s)^{\eta_s(1-\chi_s)}\left(\frac{\chi_s}{p_m}\right)^{\eta_s\chi_s}\left(\frac{1-\eta_s}{\eta_s p_l}\right)^{1-\eta_s}$$
$$\times g'\left((1-\chi_s)^{\eta_s-\chi_s\eta_s}\left(\frac{p_m}{\chi_s}\right)^{1-\eta_s\chi_s}\left(\frac{1-\eta_s}{\eta_s p_l}\right)^{1-\eta_s}\mu_s\right)$$
$$= \lim_{E\uparrow E^{m*}}\frac{\partial V_s}{\partial E}$$

Meanwhile

$$\lim_{E\downarrow E_{l*}}\frac{\partial V_s}{\partial E} = g'\left(\left(\frac{\chi_s}{p_m}\right)^{\chi_s\eta_s}(1-\chi_s)^{(1-\chi_s)\eta_s}[E_{l*}-p_l+p_m\mu_s]^{\eta_s}\right)\eta_s\left(\frac{\chi_s}{p_m}\right)^{\chi_s\eta_s}(1-\chi_s)^{(1-\chi_s)\eta_s}[E_{l*}-p_l+p_m\mu_s]^{\eta_s-1}$$

$$E_{l*}-p_l+p_m\mu_s = \frac{p_l}{1-\eta_s}-p_m\mu_s+p_m\mu_s-\frac{1-\eta_s}{1-\eta_s}p_l = \frac{\eta_s}{1-\eta_s}p_l$$

$$\lim_{E\downarrow E_{l*}} \frac{\partial V_s}{\partial E} = g'\left(\left(\frac{\chi_s}{p_m}\right)^{\chi_s\eta_s} (1-\chi_s)^{(1-\chi_s)\eta_s} \left(\frac{\eta_s}{1-\eta_s}p_l\right)^{\eta_s}\right) \eta_s \left(\frac{\chi_s}{p_m}\right)^{\chi_s\eta_s} (1-\chi_s)^{(1-\chi_s)\eta_s} \left(\frac{\eta_s}{1-\eta_s}p_l\right)^{\eta_s-1}$$

$$\begin{aligned}\lim_{E\uparrow E_{l*}} \frac{\partial V_s}{\partial E} &= (\eta_s(1-\chi_s))^{\eta_s(1-\chi_s)} \left(\frac{\eta_s\chi_s}{p_m}\right)^{\eta_s\chi_s} \left(\frac{1-\eta_s}{p_l}\right)^{1-\eta_s} \\ &\times g'\left((\eta_s(1-\chi_s))^{\eta_s(1-\chi_s)} \left(\frac{\eta_s\chi_s}{p_m}\right)^{\eta_s\chi_s} \left(\frac{1-\eta_s}{p_l}\right)^{1-\eta_s} (E_{l*}+p_m\mu_s)\right)\end{aligned}$$

$$E_{l*}+p_m\mu_s = \frac{p_l}{1-\eta_s} - p_m\mu_s + p_m\mu_s = \frac{p_l}{1-\eta_s}$$

$$\begin{aligned}\lim_{E\uparrow E_{l*}} \frac{\partial V_s}{\partial E} &= (\eta_s(1-\chi_s))^{\eta_s(1-\chi_s)} \left(\frac{\eta_s\chi_s}{p_m}\right)^{\eta_s\chi_s} \left(\frac{1-\eta_s}{p_l}\right)^{1-\eta_s} \\ &\times g'\left((\eta_s(1-\chi_s))^{\eta_s(1-\chi_s)} \left(\frac{\eta_s\chi_s}{p_m}\right)^{\eta_s\chi_s} \left(\frac{1-\eta_s}{p_l}\right)^{1-\eta_s} \frac{p_l}{1-\eta_s}\right)\end{aligned}$$

$$\begin{aligned}\lim_{E\uparrow E_{l*}} \frac{\partial V_s}{\partial E} &= (1-\chi_s)^{\eta_s(1-\chi_s)} \left(\frac{\chi_s}{p_m}\right)^{\eta_s\chi_s} \left(\frac{1-\eta_s}{p_l}\right)^{1-\eta_s} \eta_s^{\eta_s} \\ &\times g'\left((1-\chi_s)^{\eta_s(1-\chi_s)} \left(\frac{\chi_s}{p_m}\right)^{\eta_s\chi_s} \left(\frac{\eta_s p_l}{1-\eta_s}\right)^{\eta_s}\right)\end{aligned}$$

$$\begin{aligned}\lim_{E\uparrow E_{l*}} \frac{\partial V_s}{\partial E} &= \eta_s (1-\chi_s)^{\eta_s(1-\chi_s)} \left(\frac{\chi_s}{p_m}\right)^{\eta_s\chi_s} \left(\frac{\eta_s}{1-\eta_s}p_l\right)^{\eta_s-1} g'\left((1-\chi_s)^{\eta_s(1-\chi_s)} \left(\frac{\chi_s}{p_m}\right)^{\eta_s\chi_s} \left(\frac{\eta_s p_l}{1-\eta_s}\right)^{\eta_s}\right) \\ &= \lim_{E\downarrow E_{l*}} \frac{\partial V_s}{\partial E}.\end{aligned}$$

Thus, regardless of whether (82) holds, $\partial V_s/\partial E$ is continuous throughout $\mathbf{R}_+$ and strictly concave except at discrete points, so it is strictly concave and continuous everywhere.

B.5 Summary of Results for Out-of-School Households

Let us define the following. Define

$$\kappa_s^I = \mu_s^{\chi_s\eta_s} \tag{112}$$

$$\kappa_s^{II} = \mu_s^{\chi_s\eta_s} ((1-\chi_s)\eta_s)^{\eta_s(1-\chi_s)} (1-\eta_s)^{1-\eta_s} \left(\frac{1}{1-\chi_s\eta_s}\right)^{1-\chi_s\eta_s} \tag{113}$$

$$\kappa_s^{III} = \chi_s^{\chi_s\eta_s} (1-\chi_s)^{(1-\chi_s)\eta_s} \tag{114}$$

$$\kappa_s^{IV} = (\eta_s(1-\chi_s))^{\eta_s(1-\chi_s)} (\eta_s\chi_s)^{\eta_s\chi_s} (1-\eta_s)^{1-\eta_s} \tag{115}$$

If $p_l \geq \frac{1-\eta_s}{\eta_s}\frac{p_m\mu_s}{\chi_s}$, the value function is

$$V_s(E,p_l,p_m) = \begin{cases} g\left(\kappa_s^{II}\frac{E^{1-\chi_s\eta_s}}{p_l^{1-\eta_s}}\right) & E \leq \frac{1-\eta_s\chi_s}{\eta_s\chi_s}p_m\mu_s \\ g\left(\frac{\kappa_s^{IV}}{p_m^{\eta_s\chi_s}p_l^{1-\eta_s}}(E+p_m\mu_s)\right) & \frac{1-\eta_s\chi_s}{\eta_s\chi_s}p_m\mu_s < E < \frac{p_l}{1-\eta_s} - p_m\mu_s \\ g\left(\kappa_s^{III}\frac{(E-p_l+p_m\mu_s)^{\eta_s}}{p_m^{\chi_s\eta_s}}\right) & \frac{p_l}{1-\eta_s} - p_m\mu_s \leq E \end{cases} \tag{116}$$

If $p_l < \frac{1-\eta_s}{\eta_s}\frac{p_m\mu_s}{\chi_s}$, the value function is

$$V_s(E,p_l,p_m) = \begin{cases} g\left(\kappa_s^{II}\frac{E^{1-\chi_s\eta_s}}{p_l^{1-\eta_s}}\right) & E < \left[1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right]p_l \\ g(\varkappa_s^I(E-p_l)^{\eta_s(1-\chi_s)}) & \left[1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right]p_l \le E \le \frac{(1-\chi_s)}{\chi_s}p_m\mu_s + p_l \\ g\left(\kappa_s^{III}\frac{(E-p_l+p_m\mu_s)^{\eta_s}}{p_m^{\chi_s\eta_s}}\right) & \frac{(1-\chi_s)}{\chi_s}p_m\mu_s + p_l < E \end{cases} \tag{117}$$

If $p_l \ge \frac{1-\eta_s}{\eta_s}\frac{p_m\mu_s}{\chi_s}$, the marginal value function is

$$\frac{\partial V_s}{\partial E}(E,p_l,p_m) = \begin{cases} (1-\chi_s\eta_s)\frac{\kappa_s^{II}}{p_l^{1-\eta_s}}g'\left(\frac{\kappa_s^{II}E^{1-\chi_s\eta_s}}{p_l^{1-\eta_s}}\right)E^{-\chi_s\eta_s} & E \le \frac{1-\eta_s\chi_s}{\eta_s\chi_s}p_m\mu_s \\ \frac{\kappa_s^{IV}}{p_m^{\eta_s\chi_s}p_l^{1-\eta_s}}g'\left(\frac{\kappa_s^{IV}}{p_m^{\eta_s\chi_s}p_l^{1-\eta_s}}(E+p_m\mu_s)\right) & \frac{1-\eta_s\chi_s}{\eta_s\chi_s}p_m\mu_s < E < \frac{p_l}{1-\eta_s} - p_m\mu_s \\ \frac{\eta_s\kappa_s^{III}}{p_m^{\chi_s\eta_s}}g'\left(\kappa_s^{III}\frac{(E-p_l+p_m\mu_s)^{\eta_s}}{p_m^{\chi_s\eta_s}}\right)(E-p_l+p_m\mu_s)^{\eta_s-1} & \frac{p_l}{1-\eta_s} - p_m\mu_s \le E \end{cases} \tag{118}$$

Note that Regimes I and II are only applicable if $\mu_s > 0$ so remittances are a luxury. If $\mu_s = 0$, remittances are a necessity that satisfy an Inada condition, so $m = 0$ is only possible if $E = 0$.

If $p_l < \frac{1-\eta_s}{\eta_s}\frac{p_m\mu_s}{\chi_s}$, the marginal value function is

$$\frac{\partial V_s}{\partial E}(E,p_l,p_m) = \begin{cases} (1-\chi_s\eta_s)\frac{\kappa_s^{II}}{p_l^{1-\eta_s}}g'\left(\frac{\kappa_s^{II}E^{1-\chi_s\eta_s}}{p_l^{1-\eta_s}}\right)E^{-\chi_s\eta_s} & E < \left[1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right]p_l \\ \eta_s(1-\chi_s)\kappa_s^I g'(\varkappa_s^I(E-p_l)^{\eta_s(1-\chi_s)})(E-p_l)^{\eta_s(1-\chi_s)-1} & \left[1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right]p_l \le E \le \frac{(1-\chi_s)}{\chi_s}p_m\mu_s + p_l \\ \frac{\eta_s\kappa_s^{III}}{p_m^{\chi_s\eta_s}}g'\left(\kappa_s^{III}\frac{(E-p_l+p_m\mu_s)^{\eta_s}}{p_m^{\chi_s\eta_s}}\right)(E-p_l+p_m\mu_s)^{\eta_s-1} & \frac{(1-\chi_s)}{\chi_s}p_m\mu_s + p_l < E \end{cases} \tag{119}$$

If $p_l \ge \frac{1-\eta_s}{\eta_s}\frac{p_m\mu_s}{\chi_s}$, the consumption function is

$$c(E,p_l,p_m;s) = \begin{cases} \frac{(1-\chi_s)\eta_s}{1-\chi_s\eta_s}E & E \le \frac{1-\eta_s\chi_s}{\eta_s\chi_s}p_m\mu_s \\ \eta_s(1-\chi_s)[E+p_m\mu_s] & \frac{1-\eta_s\chi_s}{\eta_s\chi_s}p_m\mu_s < E < \frac{p_l}{1-\eta_s} - p_m\mu_s \\ (1-\chi_s)[E-p_l+p_m\mu_s] & \frac{p_l}{1-\eta_s} - p_m\mu_s \le E \end{cases} \tag{120}$$

If $p_l < \frac{1-\eta_s}{\eta_s}\frac{p_m\mu_s}{\chi_s}$, the consumption function is

$$c(E,p_l,p_m;s) = \begin{cases} \frac{(1-\chi_s)\eta_s}{1-\chi_s\eta_s}E & E < \left[1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right]p_l \\ E-p_l & \left[1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right]p_l \le E \le \frac{(1-\chi_s)}{\chi_s}p_m\mu_s + p_l \\ (1-\chi_s)[E-p_l+p_m\mu_s] & \frac{(1-\chi_s)}{\chi_s}p_m\mu_s + p_l < E \end{cases} \tag{121}$$

If $p_l \ge \frac{1-\eta_s}{\eta_s}\frac{p_m\mu_s}{\chi_s}$, the leisure function is

$$l(E,p_l,p_m;s) = \begin{cases} \frac{(1-\eta_s)E}{(1-\chi_s\eta_s)p_l} & E \le \frac{1-\eta_s\chi_s}{\eta_s\chi_s}p_m\mu_s \\ (1-\eta_s)\frac{E+p_m\mu_s}{p_l} & \frac{1-\eta_s\chi_s}{\eta_s\chi_s}p_m\mu_s < E < \frac{p_l}{1-\eta_s} - p_m\mu_s \\ 1 & \frac{p_l}{1-\eta_s} - p_m\mu_s \le E \end{cases} \tag{122}$$

If $p_l < \frac{1-\eta_s}{\eta_s}\frac{p_m\mu_s}{\chi_s}$, the leisure function is

$$l(E,p_l,p_m;s) = \begin{cases} \frac{(1-\eta_s)E}{(1-\chi_s\eta_s)p_l} & E < \left[1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right]p_l \\ 1 & \left[1+(1-\chi_s)\frac{\eta_s}{1-\eta_s}\right]p_l \le E \end{cases} \tag{123}$$

If $p_l \geq \frac{1-\eta_s}{\eta_s}\frac{p_m\mu_s}{\chi_s}$, the remittance function is

$$m(E, p_l, p_m; s) = \begin{cases} 0 & E \leq \frac{1-\eta_s\chi_s}{\eta_s\chi_s}p_m\mu_s \\ \frac{\eta_s\chi_s}{p_m}E + (\eta_s\chi_s - 1)\mu_s & \frac{1-\eta_s\chi_s}{\eta_s\chi_s}p_m\mu_s < E < \frac{p_l}{1-\eta_s} - p_m\mu_s \\ \frac{\chi_s}{p_m}(E - p_l) - (1-\chi_s)\mu_s & \frac{p_l}{1-\eta_s} - p_m\mu_s \leq E \end{cases} \tag{124}$$

If $p_l < \frac{1-\eta_s}{\eta_s}\frac{p_m\mu_s}{\chi_s}$, the remittance function is

$$m(E, p_l, p_m; s) = \begin{cases} 0 & E \leq \frac{(1-\chi_s)}{\chi_s}p_m\mu_s + p_l \\ \frac{\chi_s}{p_m}(E - p_l) - (1-\chi_s)\mu_s & \frac{(1-\chi_s)}{\chi_s}p_m\mu_s + p_l < E \end{cases} \tag{125}$$

Suppose that g is CRRA with risk aversion γ. Then we have an analytic expression for the inverse marginal value function, which we will denote by q.

If $p_l \geq \frac{1-\eta_s}{\eta_s}\frac{p_m\mu_s}{\chi_s}$, the marginal value function is

$$\frac{\partial V_s}{\partial E}(E, p_l, p_m) = \begin{cases} (1-\chi_s\eta_s)\frac{\kappa_s^{II}}{p_l^{1-\eta_s}}\left(\frac{\kappa_s^{II}E^{1-\chi_s\eta_s}}{p_l^{1-\eta_s}}\right)^{-\gamma}E^{-\chi_s\eta_s} & E \leq \frac{1-\eta_s\chi_s}{\eta_s\chi_s}p_m\mu_s \\ \frac{\kappa_s^{IV}}{p_m^{\eta_s\chi_s}p_l^{1-\eta_s}}\left(\frac{\kappa_s^{IV}}{p_m^{\eta_s\chi_s}p_l^{1-\eta_s}}(E + p_m\mu_s)\right)^{-\gamma} & \frac{1-\eta_s\chi_s}{\eta_s\chi_s}p_m\mu_s < E < \frac{p_l}{1-\eta_s} - p_m\mu_s \\ \frac{\eta_s\kappa_s^{III}}{p_m^{\chi_s\eta_s}}\left(\kappa_s^{III}\frac{(E-p_l+p_m\mu_s)^{\eta_s}}{p_m^{\chi_s\eta_s}}\right)^{-\gamma}(E - p_l + p_m\mu_s)^{\eta_s-1} & \frac{p_l}{1-\eta_s} - p_m\mu_s \leq E \end{cases}$$

Let us write the value function as

$$V = g(\kappa p_l^{\nu_l} p_m^{\nu_m}(E + \Delta(p_l, p_m))^{\nu}).$$

$$\frac{\partial V}{\partial E} = g'(\kappa p_l^{\nu_l} p_m^{\nu_m}(E + \Delta(p_l, p_m))^{\nu})\kappa\nu p_l^{\nu_l} p_m^{\nu_m}(E + \Delta(p_l, p_m))^{\nu-1}.$$

If risk aversion is γ,

$$\begin{aligned} \frac{\partial V}{\partial E} &= (\kappa p_l^{\nu_l} p_m^{\nu_m}(E + \Delta(p_l, p_m))^{\nu})^{-\gamma}\kappa\nu p_l^{\nu_l} p_m^{\nu_m}(E + \Delta(p_l, p_m))^{\nu-1} \\ &= \nu(\kappa p_l^{\nu_l} p_m^{\nu_m})^{1-\gamma}(E + \Delta(p_l, p_m))^{(1-\gamma)\nu-1}. \end{aligned}$$

Let x be the marginal value.

$$x = \nu(\kappa p_l^{\nu_l} p_m^{\nu_m})^{1-\gamma}(E + \Delta(p_l, p_m))^{(1-\gamma)\nu-1}$$

$$(E + \Delta(p_l, p_m))^{(1-\gamma)\nu-1} = \left(\nu^{-1}\kappa^{\gamma-1}p_l^{\nu_l(\gamma-1)}p_m^{\nu_m(\gamma-1)}x\right)$$

$$E = \left(\nu^{-1}\kappa^{\gamma-1}p_l^{\nu_l(\gamma-1)}p_m^{\nu_m(\gamma-1)}x\right)^{\frac{1}{(1-\gamma)\nu-1}} - \Delta(p_l, p_m)$$

B.6 Households in School

The problem for a high-educated household while still in school is simplified relative to the general problem.

$$V_e(E, p_m) = \max u_2(c, l_e, m)$$

subject to

$$\begin{aligned} c + p_m m &= E. \\ c, m &\geq 0 \end{aligned}$$

This has first-order conditions

$$\frac{\partial u_2}{\partial c}(c, l_e, m) \geq p_m \frac{\partial u_2}{\partial m}(c, l_e, m)$$

with inequality only if $m = 0$.

$$u_2(c, l_e, m) = g\left((c^{1-\chi_2}(m+\mu_2)^{\chi_2})^{\eta_2} l_e^{1-\eta_2}\right)$$

$$(1-\chi_2)\eta_2 c^{(1-\chi_2)\eta_2 - 1}(m+\mu_2)^{\chi_2\eta_2} l_e^{1-\eta_2} \geq p_m \chi_2 \eta_2 c^{(1-\chi_2)\eta_2}(m+\mu_2)^{\chi_2\eta_2-1} l_e^{1-\eta_2}$$

$$\frac{1-\chi_2}{c} \geq p_m \frac{\chi_2}{m+\mu_2}.$$

Thus

$$c \leq \frac{1-\chi_2}{\chi_2} p_m (m+\mu_2) \tag{126}$$

with inequality only if $m = 0$.

B.6.1 Regime V: No Remittances in School

In this case, $c = E$ and we need

$$E = c \leq \frac{1-\chi_2}{\chi_2} p_m \mu_2. \tag{127}$$

Thus

$$V_e(E, p_m) = g\left(E^{\eta_2(1-\chi_2)} \mu_2^{\eta_2\chi_2} l_e^{1-\eta_2}\right). \tag{128}$$

$$\frac{\partial V_e}{\partial E} = \eta_2(1-\chi_2) g'\left(E^{\eta_2(1-\chi_2)} \mu_2^{\eta_2\chi_2} l_e^{1-\eta_2}\right) E^{\eta_2(1-\chi_2)-1} \mu_2^{\eta_2\chi_2} l_e^{1-\eta_2} \tag{129}$$

B.6.2 Regime VI: Remittances in School

In this case, we have

$$\begin{aligned} c + p_m m &= E \\ c &= \frac{1-\chi_2}{\chi_2} p_m (m+\mu_2) \end{aligned}$$

Thus

$$\left(1 + \frac{1-\chi_2}{\chi_2}\right) p_m m + \frac{1-\chi_2}{\chi_2} p_m \mu_2 = E$$

$$\frac{p_m m}{\chi_2} + \frac{1-\chi_2}{\chi_2} p_m \mu_2 = E$$

$$m + (1-\chi_2)\mu_2 = \frac{\chi_2}{p_m} E$$

$$m = \frac{\chi_2}{p_m} E - (1-\chi_2)\mu_2 \tag{130}$$

$$
\begin{aligned}
m + \mu_2 &= \frac{\chi_2}{p_m}(E + p_m\mu_2) \\
c &= (1-\chi_2)(E + p_m\mu_2)
\end{aligned} \tag{131}
$$

This is valid as long as $m \geq 0$. Thus

$$
\begin{aligned}
\frac{\chi_2}{p_m} E &\geq (1-\chi_2)\mu_2 \\
E &\geq \frac{1-\chi_2}{\chi_2} p_m \mu_2
\end{aligned}
$$

$$
\begin{aligned}
V_e(E,p_m) &= g\left(((1-\chi_2)(E+p_m\mu_2))^{\eta_2(1-\chi_2)} \left(\frac{\chi_2}{p_m}(E+p_m\mu_2)\right)^{\eta_2\chi_2} l_e^{1-\eta_2} \right) \\
V_e(E,p_m) &= g\left((1-\chi_2)^{\eta_2(1-\chi_2)} \left(\frac{\chi_2}{p_m}\right)^{\eta_2\chi_2} l_e^{1-\eta_2} (E+p_m\mu_2)^{\eta_2} \right)
\end{aligned} \tag{132}
$$

$$
\frac{\partial V_e(E,p_m)}{\partial E} = \eta_2 g'\left((1-\chi_2)^{\eta_2(1-\chi_2)} \left(\frac{\chi_2}{p_m}\right)^{\eta_2\chi_2} l_e^{1-\eta_2}(E+p_m\mu_2)^{\eta_2} \right) (1-\chi_2)^{\eta_2(1-\chi_2)} \left(\frac{\chi_2}{p_m}\right)^{\eta_2\chi_2} l_e^{1-\eta_2}(E+p_m\mu_2)^{\eta_2-1} \tag{133}
$$

Note that

$$
\begin{aligned}
\lim_{E\downarrow \frac{1-\chi_2}{\chi_2}p_m\mu_2} V_e(E,p_m) &= g\left((1-\chi_2)^{\eta_2(1-\chi_2)} \left(\frac{\chi_2}{p_m}\right)^{\eta_2\chi_2} l_e^{1-\eta_2} \left(\frac{p_m\mu_2}{\chi_2}\right)^{\eta_2} \right) \\
&= g\left(\left(\frac{1-\chi_2}{\chi_2}p_m\right)^{\eta_2(1-\chi_2)} l_e^{1-\eta_2}\mu_2^{\eta_2} \right)
\end{aligned}
$$

$$
\begin{aligned}
\lim_{E\uparrow \frac{1-\chi_2}{\chi_2}p_m\mu_2} V_e(E,p_m) &= g\left(\left(\frac{1-\chi_2}{\chi_2}p_m\mu_2\right)^{\eta_2(1-\chi_2)} \mu_2^{\eta_2\chi_2} l_e^{1-\eta_2} \right) \\
&= g\left(\left(\frac{1-\chi_2}{\chi_2}p_m\right)^{\eta_2(1-\chi_2)} \mu_2^{\eta_2} l_e^{1-\eta_2} \right) = \lim_{E\uparrow \frac{1-\chi_2}{\chi_2}p_m\mu_2} V_e(E,p_m)
\end{aligned}
$$

Thus V_e is continuous and $E^{\eta_2(1-\chi_2)}\mu_2^{\eta_2\chi_2}l_e^{1-\eta_2}$ is also continuous since g is invertible.

$$
\begin{aligned}
\lim_{E\downarrow \frac{1-\chi_2}{\chi_2}p_m\mu_2} \frac{\partial V_e(E,p_m)}{\partial E} &= \lim_{E\downarrow \frac{1-\chi_2}{\chi_2}p_m\mu_2} g'\left(E^{\eta_2(1-\chi_2)}\mu_2^{\eta_2\chi_2} l_e^{1-\eta_2} \right) \eta_2 (1-\chi_2)^{\eta_2(1-\chi_2)} \\
&\quad \times \left(\frac{\chi_2}{p_m}\right)^{\eta_2\chi_2} l_e^{1-\eta_2} \left(\frac{1-\chi_2}{\chi_2}p_m\mu_2 + p_m\mu_2\right)^{\eta_2-1} \\
&= g'\left(\left(\frac{1-\chi_2}{\chi_2}p_m\right)^{\eta_2(1-\chi_2)} l_e^{1-\eta_2}\mu_2^{\eta_2} \right) \eta_2 (1-\chi_2)^{\eta_2(1-\chi_2)} \left(\frac{\chi_2}{p_m}\right)^{\eta_2\chi_2} l_e^{1-\eta_2} \left(\frac{p_m\mu_2}{\chi_2}\right)^{\eta_2-1} \\
&= g'\left(\left(\frac{1-\chi_2}{\chi_2}p_m\right)^{\eta_2(1-\chi_2)} l_e^{1-\eta_2}\mu_2^{\eta_2} \right) \eta_2 (1-\chi_2)^{\eta_2(1-\chi_2)} \left(\frac{\chi_2}{p_m}\right)^{\eta_2\chi_2+1-\eta_2} l_e^{1-\eta_2}\mu_2^{\eta_2-1}
\end{aligned}
$$

$$\lim_{E\uparrow \frac{1-\chi_2}{\chi_2}p_m\mu_2} \frac{\partial V_e(E,p_m)}{\partial E} = g'\left(\left(\frac{1-\chi_2}{\chi_2}p_m\right)^{\eta_2(1-\chi_2)}\mu_2^{\eta_2}l_e^{1-\eta_2}\right)\eta_2(1-\chi_2)\left(\frac{1-\chi_2}{\chi_2}p_m\mu_2\right)^{\eta_2(1-\chi_2)-1}\mu_2^{\eta_2\chi_2}l_e^{1-\eta_2}$$
$$= g'\left(\left(\frac{1-\chi_2}{\chi_2}p_m\right)^{\eta_2(1-\chi_2)}\mu_2^{\eta_2}l_e^{1-\eta_2}\right)\eta_2(1-\chi_2)^{\eta_2(1-\chi_2)}\left(\frac{p_m\mu_2}{\chi_2}\right)^{\eta_2(1-\chi_2)-1}\mu_2^{\eta_2\chi_2}l_e^{1-\eta_2}$$
$$= g'\left(\left(\frac{1-\chi_2}{\chi_2}p_m\right)^{\eta_2(1-\chi_2)}\mu_2^{\eta_2}l_e^{1-\eta_2}\right)\eta_2(1-\chi_2)^{\eta_2(1-\chi_2)}\left(\frac{\chi_2}{p_m}\right)^{\eta_2\chi_2+1-\eta_2}\mu_2^{\eta_2-1}l_e^{1-\eta_2}$$
$$= \lim_{E\downarrow \frac{1-\chi_2}{\chi_2}p_m\mu_2} \frac{\partial V_e(E,p_m)}{\partial E}$$

Thus $\frac{\partial V_e(E,p_m)}{\partial E}$ is also continuous. The threshold value of the marginal value function is

$$\frac{\partial V_e(E_*,p_m)}{\partial E} = \left(\left(\frac{1-\chi_2}{\chi_2}p_m\right)^{\eta_2(1-\chi_2)}\mu_2^{\eta_2}l_e^{1-\eta_2}\right)^{-\gamma}\eta_2(1-\chi_2)^{\eta_2(1-\chi_2)}\left(\frac{\chi_2}{p_m}\right)^{\eta_2\chi_2+1-\eta_2}\mu_2^{\eta_2-1}l_e^{1-\eta_2}$$
$$= \left(\left(\frac{1-\chi_2}{\chi_2}p_m\right)^{\eta_2(1-\chi_2)}\mu_2^{\eta_2}l_e^{1-\eta_2}\right)^{-\gamma}\eta_2\left(\frac{1-\chi_2}{\chi_2}p_m\right)^{\eta_2(1-\chi_2)}\mu_2^{\eta_2}l_e^{1-\eta_2}\frac{\chi_2}{p_m\mu_2}$$
$$= \eta_2\left(\left(\frac{1-\chi_2}{\chi_2}p_m\right)^{\eta_2(1-\chi_2)}\mu_2^{\eta_2}l_e^{1-\eta_2}\right)^{1-\gamma}\frac{\chi_2}{p_m\mu_2}$$
$$= \eta_2\left(\left(\frac{1-\chi_2}{\chi_2}\right)^{\eta_2(1-\chi_2)}\mu_2^{\eta_2}l_e^{1-\eta_2}\right)^{1-\gamma}\frac{\chi_2}{\mu_2}p_m^{(1-\gamma)\eta_2(1-\chi_2)-1}$$

C Solution Algorithm

Equilibria are computed by the following algorithm:

1. The parameters of the value function in (116)-(117), (128), and (132) are computed so that we have analytic function calls for $V(E,p_l,p_m)$, $\partial V/\partial E(E,p_l,p_m)$, $(\partial V/\partial E)^{-1}(\lambda,p_l,p_m)$, $V_e(E,p_m)$, $\partial V_e/\partial E(E,p_m)$, and $(\partial V_e/\partial E)^{-1}(\lambda,p_m)$.

2. Define a state vector $\theta = (K, \frac{n_2}{n_1}, N_0, N_1, N_2, M)$. We guess at an initial state vector.

3. Compute the factor prices. Depending on whether we hold the tax rates or the Social Security replacement rates fixed, compute the other rates so the government budget constraint (21) holds, keeping the floating rate vector proportional to the initial calibration of that 3-dimensional vector. We also compute the Social Security benefits B_s and the endowment (and its lifetime present value) for each type of household.[13] At this point, the leisure price p_l will be known for each household type and age ($p_m = 1+\tau_m$ is constant), so we also compute the thresholds dividing the regimes for the inverse marginal value function $(\partial V/\partial E)^{-1}$.

[13]If either the benefits or the rates are negative, we set them to zero. If an income tax rate is above 95%, we set it to 95%. In equilibrium, these bounds cannot be violated, but they may be violated at some iteration of the algorithm.

4. Solve the intertemporal problem. In the following, we suppress the dependence on p_l and p_m, which are known at this point.

   - First assume the borrowing constraints do not hold. Solve for E_0 such that the lifetime budget constraint holds, where
   $$E_{t+1} = \left(\frac{\partial V}{\partial E}\right)^{-1} \left(\beta R \frac{\partial V}{\partial E}(E_t)\right) \tag{134}$$
   for $t = 0, 1$.
   - If the borrowing constraints are not satisfied by that solution, assume the household is constrained when young. E_0 is determined by the borrowing constraint. Solve for E_1 that satisfies the lifetime budget constraint with E_2 given by (134).
   - If the borrowing constraint at t constraint at $t = 1$ or the Euler inequality
   $$\frac{\partial V}{\partial E}(E_t) \geq \beta R \frac{\partial V}{\partial E}(E_{t+1}) \tag{135}$$
   does not hold at $t = 0$, assume the household is constrained at all ages. E_0, E_1, and E_2 will be determined.by the borrowing constraints.
   - If the Euler inequalities (135) do not hold at $t = 0$ or $t = 1$, assume the household is only constrained in middle age. E_2 is determined by the borrowing constraint. Solve for E_0 that satisfies the lifetime budget constraint with E_1 given by (134).
   - One of these four cases must hold if the program is coded correctly.

5. Given the $E_t(s)$, compute $c_t(s)$, $l_t(s)$, and $m_t(s)$ for $s = 0, 1, 2$ and $t = 0, 1, 2$. Aggregate the labor supplies, remittances, and the capital stock.

6. Compute the equivalent variation Δ that satisfies
$$\sum_{t=0}^{2} \beta^t u_1((1+\Delta)c_t(1), l_t(1), m_t(1)) = \sum_{t=0}^{2} \beta^t u_2(c_t(2), l_t(2), m_t(2)) \tag{136}$$
and adjust the population ratio
$$\frac{n_2}{n_1} \to \exp(0.1\Delta)\frac{n_2}{n_1}. \tag{137}$$

7. Go back to step 3 until the change in the capital stock and Δ are both sufficiently close to zero.

References

[1] Bagchi, Shantanu and James A. Feigenbaum, (2014), "Is Smoking a Fiscal Good?" *Review of Economic Dynamics* 17: 170-190.

[2] Borjas, George J., (2016), "The Wage Impact of the *Marielitos*: A Reappraisal," Forthcoming in *Industrial and Labor Relations Review*.

[3] Casarico, Alessandra, Giovanni Facchini, and Tommaso Frattini, (2016), "What Drives the Legalization of Immigrants? Evidence from IRCA," Working Paper.

[4] Feigenbaum, James, (2015), "Transition Dynamics of a Mass Deportation," Working Paper.

[5] Krause, Alan, (2017), "On Redistributive Taxation Under the Threat of High-Skill Emigration," *Social Choice and Welfare* DOI 10.1007/s00355-017-1038-8.